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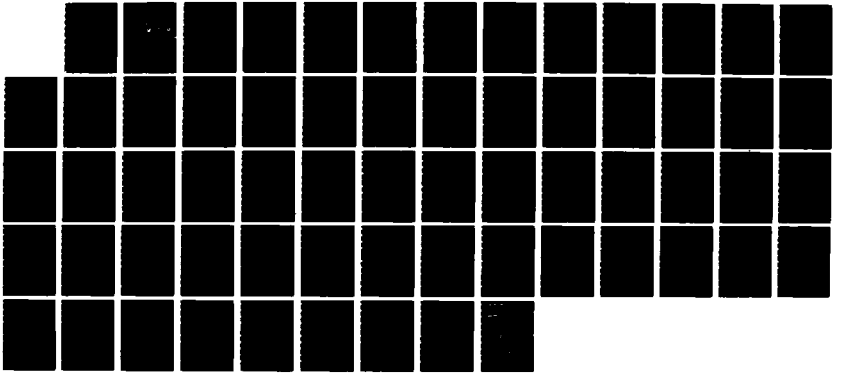

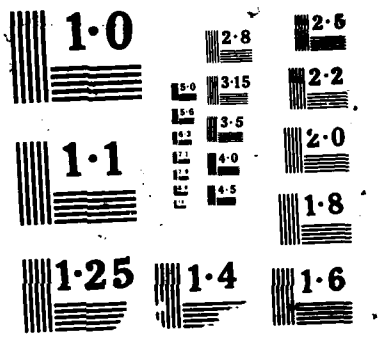
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NAVAL POSTGRADUATE SCHOOL

Monterey, California

CONTRACTOR REPORT

AMBIENT SCATTERING FROM RING-SYMMETRIC

SPACECRAFT EXHAUST PLUME

by

Joseph Falcovitz

April 1987

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The Pentagon
Washington, DC 20301-7100

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ABSTRACT

We present a first-collision model for the evaluation of return flux from the exhaust plume of a ring-symmetric HF/DF laser in LEO, generated by an incident flux of ambient molecules traveling at orbital speed. The steady plume is bounded by a pair of lip-centered rarefaction fans, and unless spacecraft attitude enables incident air molecules to reach the plume through the cavitation regions that extend beyond these fans, the spacecraft is shielded from ambient scattering by its own plume. Assuming hard-spheres collisions, the first-collision model is given by a simple closed-form expression that can be regarded as a source term for scattered exhaust molecules. This source term is integrated numerically throughout the fan, yielding the flux arriving at some surface "target point". Quantitatively, it is shown that for a typical HF/DF laser exhaust the contamination level generated by ambient scattering is not significant. It was found that the maximum return flux of HF+DF constitutes about 2% of the incident ambient flux; this ratio will be nearly constant for LEO altitudes. The value of this flux ratio is shown to be dependent on the molecular collision model; it may change upon replacing the hard-spheres approximation by a more realistic collision model. A possible modification of spacecraft charging by the exhaust was examined, including production of HF^- and DF^-. The only significant effect seemed to be shadowing of the downstream half of the spacecraft at oblique orbital attitudes.

ACKNOWLEDGEMENTS

This work is part of a study involving gas dynamics of exhaust plumes from spacecrafts. It was conducted under the cognizance of Distinguished Professor Allen E. Fuhs, who initiated this research program at the Naval Postgraduate School. I wish to thank Professor Fuhs for his inspiring guidance and deeply appreciate his continued support.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

EMPTY PAGE

1. INTRODUCTION

This presentation is part of a study on the gas dynamics of ring-symmetric exhaust plumes in space, conducted at the Naval Postgraduate School in Monterey. A ring-symmetric jet has zero thrust, which makes it suitable as an exhaust configuration for various open loop power plants designed to produce high power for relatively short durations. One such system is an envisioned space-based chemical laser, shown schematically in Fig. 1-1. In the case of a chemical laser, a ring-symmetric configuration would also enable the laser radiation to emerge in the form of an axisymmetric beam.

The exhaust nozzle should be designed to bring the outgoing flow to a supersonic speed at the nozzle exit surface. The near field of a free jet is then composed of an inner core bounded by a pair of ring-symmetric rarefaction fans centered at the nozzle lips (Fig. 1-1). Beyond the limiting characteristic surface of the centered rarefaction waves (CRW), a near-vacuum condition prevails. For the purpose of continuum gas dynamic analysis, we assume it is a perfect vacuum.

An earth orbiting vehicle is subject to an oncoming stream of ambient molecules at a speed of $U_A \approx 8$ (km/sec), in a direction depending upon its orientation relative to the orbital velocity vector. This speed is sufficiently high to cause backscattering of exhaust molecules (see schematic description in Fig. 1-2) moving at speeds appropriate to chemical combustion (about 2 to 4 km s). However, large exhaust plumes, having achieved stationary flow, may be sufficiently dense at their outer fringes to effectively trap and entrain all oncoming ambient molecules. Thus, ambient scattering may be significant only in selected ranges of attitude angles, at which ambient molecules can reach the vicinity of the spacecraft by traveling almost collisionlessly through cavitation regions. Exhaust molecules that may be "candidates" for ambient scattering will hence come from plume segments flanked by cavitation regions. The contribution of ambient scattering to contamination will thus be highly dependent upon spacecraft geometry and orientation. This may well affect spacecraft design and operating procedures.

The purpose of this report is to present a first-collision model for estimating the flux of exhaust molecules backscattered from the fringes of the plume by ambient molecules, along with results of sample flux computations performed on a typical HF DF laser exhaust configuration. The flow field throughout the plume is assumed to be governed by the equations of continuum gas dynamics. In principle, the flow could be obtained by solving the governing equations, i.e., the equations for stationary isentropic flow in two-dimensional axisymmetric coordinates. In practice, this is normally

accomplished by integrating the flow equations in characteristic form, using some finite difference scheme (method-of-characteristics). We have performed such computations, but given the complexity of applying them to the subsequent integration of ambient scattering flux (due to the need for two-dimensional interpolations from an irregular solution grid), we opted for a different alternative : a closed-form approximation to the ring-symmetric CRW, based on an analytic expression for flow variables along characteristic lines that fan out from the nozzle lip.

The plan of this report is as follows. In Ch. 2 we outline the approximation to the ring-symmetric CRW and present some computation results that demonstrate its accuracy. In Ch. 3 we describe the first-collision model and the 3-D spatial integration scheme for computing the flux arriving at the cylindrical spacecraft. In Ch. 4 some results of backscattered flux of corrosive molecules (HF + DF), showing flux variation with target point location ($\mathbf{X}_s$) and attitude angles (ψ_A, φ_A) are presented. In Ch. 5 we take up the subject of spacecraft charging, using results of ambient scattering to assess the effect of laser exhaust on spacecraft charging. This is followed by concluding remarks in Ch. 6 and a list of references in Ch. 7. A concise description of the flux computation code "AMB" is given in Appendix A, followed by the code listing.

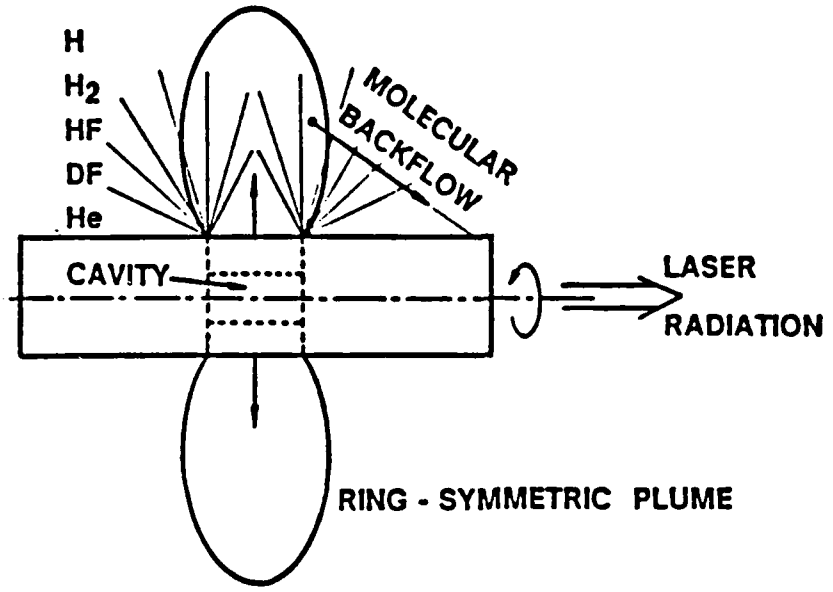

Figure 1-1. Ring-Symmetric HF, DF Laser Exhaust Plume.

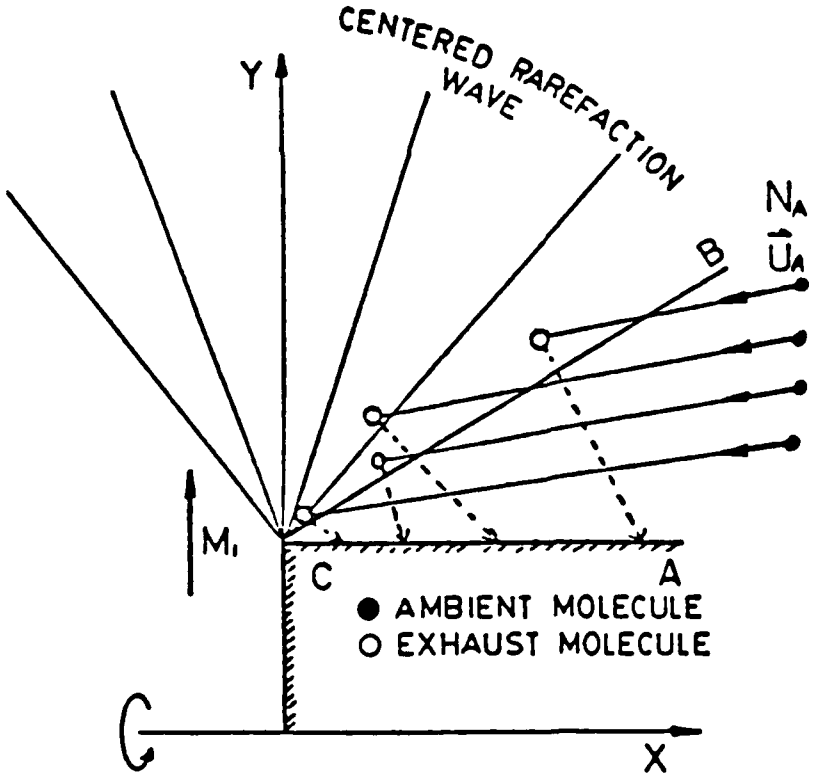

Figure 1-2. Schematic Description of Ambient Scattering. The Cavitation Region is Bounded by Lines CA and CB.

2. COMPUTATION OF THE PLUME FLOW FIELD

Most ambient molecules entering the CRW that flanks the exhaust plume are stopped within several mean free paths from their point of entry. A quantitative estimate of ambient back-scattering would thus depend on the flow field at the outer (hypersonic) fringes of the lip-centered CRW. Even though the flow in those regions is generally past the surface of continuum breakdown, the density there is reasonably well approximated by the continuum flow field, as demonstrated by Bird's Monte-Carlo simulation of a Prandtl-Meyer expansion to vacuum [1] . The evaluation of ambient scattering thus calls for an ancillary computational procedure capable of rendering the continuum flow field at a large number of points in the ring-symmetric CRW of an exhaust plume. This method was described in a recent report [2] . Here we just outline the key ideas and main results of this approximation method.

Our analytic approximation to a ring-symmetric CRW is formulated as follows. In a planar CRW (Prandtl-Meyer flow) all flow variables are uniform along the characteristic lines that fan out from the corner (we assume they are the C^+ family). In the ring-symmetric case the flow near the corner approaches asymptotically a corresponding planar CRW flow, which we term the *associate* CRW. However, the gradients along C^+ characteristics at the corner of a ring-symmetric CRW do not vanish as in a planar CRW. The key idea is thus : evaluate flow gradients in C^+ directions at the corner, then use them to extrapolate the associate CRW along C^+ lines to a finite distance from the corner. The extrapolation is a nonlinear function of the radial coordinate y , chosen so that the ensuing expression conforms exactly to the flow at the leading (exit) characteristic $C^+(\beta_1)$. Omitting all details of the analysis, the resulting approximation is presented as the following power-law :

$$f(\alpha,\beta) = f(0,\beta)\left[y(\alpha,\beta)/y(0,\beta)\right]^{\delta(0,\beta)} \qquad (2\text{-}1)$$

where f is the streamtube area ratio for isentropic flows ($f=1$ at a sonic point), β is the Mach number of a particular characteristic line at the corner, α is a coordinate along the $C^+(\beta)$ characteristic line ($\alpha=0$ at the corner), and y is the radial coordinate of a point on the characteristic line $C^+(\beta)$. The Mach number at point (α,β) is readily determined from $f(\alpha,\beta)$ using the standard relation between area ratio and Mach number [3] . A closed-form expression for $\delta(0,\beta)$ was developed but is not given here; instead, this function is shown in Fig. 2-1. We note that δ approaches the asymptotic value of $2/(3-\gamma)$ as β increases to infinity, and that generally $1 < \delta(0,\beta) < 2$ so that streamtubes diverge at a rate intermediate between that of cylindrical and spherical expansion flows.

Clearly, in an isentropic flow all thermodynamic variables, and in particular density, can be evaluated from $\mathbf{f}$. This approximation is readily applied to the hypersonic portions of a ring-symmetric CRW since it turns out that characteristic lines are nearly straight there, which means that the characteristic line $C^+(\beta)$ passing through a given point can be readily determined. As a demonstration of the degree of accuracy obtainable from this approximation, we show in Fig. 2-2 the variation of Mach number along a characteristic line in the ring-symmetric CRW, compared with an accurate method-of-characteristics computation. This comparison demonstrates that the analytic approximation is reasonably accurate to nearly ten corner-radii away from the corner.

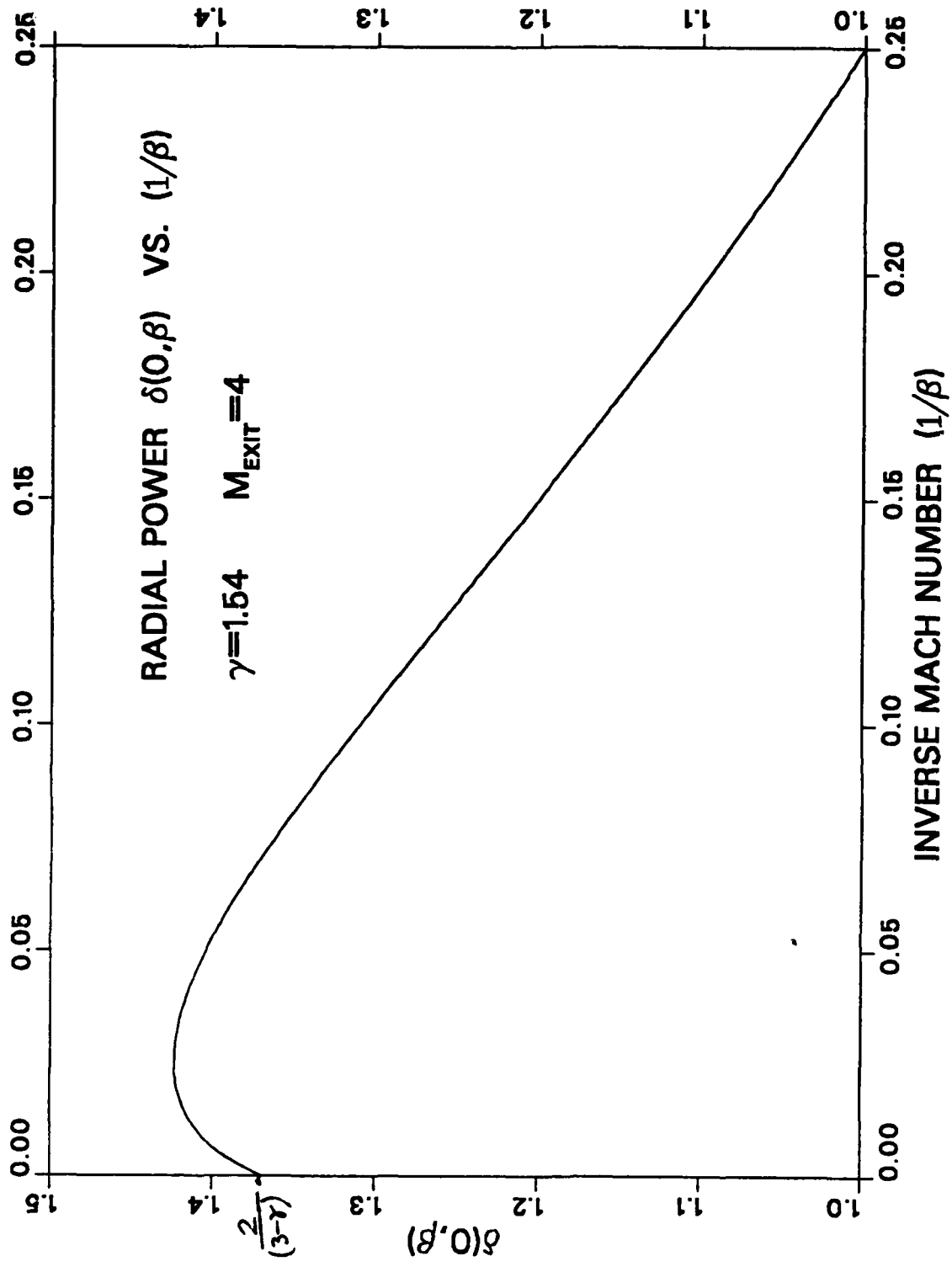

Figure 2-1. Power $\delta(0,\beta)$ for the power-law Approximation.

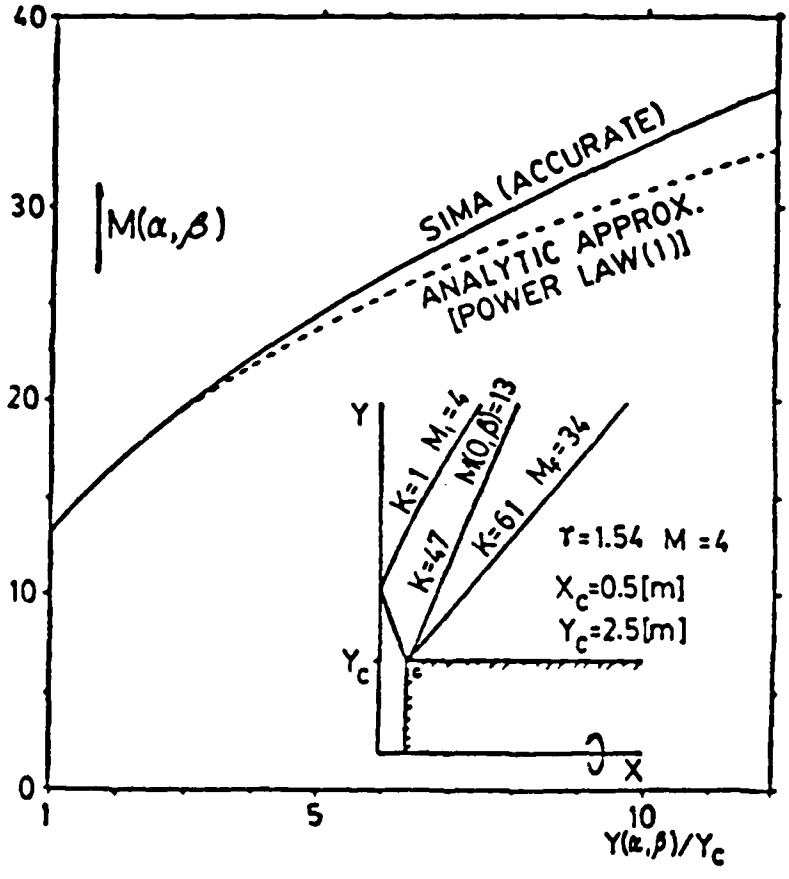

Figure 2-2. Variation of Mach Number along Characteristic Line $\beta = 13$.

3. AMBIENT SCATTERING

When a rocket or laser exhaust is released into space from an earth-orbiting spacecraft, it encounters an oncoming stream of ambient molecules flowing at the orbital speed of $U_A \approx 8$ (km/sec). At altitudes higher than 200 (km), the air air mean free path exceeds 250 (m), so that it is considerably larger than almost any spacecraft. Consequently, ambient molecules would hardly be subjected to a shock transition prior to their impact at the spacecraft or exhaust plume. In this chapter we describe the formulation of the first-collision model in Section 3.1 and then proceed to present the derivation of the flux integration scheme for hard-sphere collisions in Section 3.2.

3.1 First Collision Model

The highest ambient number density that we consider for earth-orbiting spacecrafts is $N_A = 1 \times 10^{16}$ (m^{-3}), which roughly corresponds to Sunspot Maximum at 200 (km) [4] . The typical laser exhaust (Table 4-1) would reach a number density of about 2×10^{19} (m^{-3}) at the very high Mach number of 30. Hence, ambient flux constitutes just a slight perturbation to the near-field portion of a typical laser exhaust plume. Obviously, ambient molecules that penetrate the plume, would subsequently be entrained by the main flow. But how far do they penetrate? And would exhaust molecules scattered by them reach the spacecraft? In seeking answers to these questions, we are led to some interesting observations concerning ambient scattering.

Consider the HF laser depicted in Fig. 1-1. The spacecraft diameter is 5 (m) and the centrally located ring-symmetric nozzle is 1 (m) wide. Typical operating conditions (Table 4-1) are assumed. They are based on some experimental HF DF laser studies conducted at TRW [5,6] . Suppose that the spacecraft axis is normal to the orbital velocity vector (normal incidence). Let the plane of incidence be the plane defined by the intersection of the spacecraft axis with the orbital velocity vector. The probability that an ambient molecule traveling in the plane of incidence would reach the spacecraft collisionlessly is $\exp(-\eta)$, where η is its expected number of collisions with exhaust molecules. We define the number η as "molecular thickness", in analogy to "optical thickness". So in order to determine the extent to which ambient molecules at normal incidence reach the spacecraft, we seek the distribution of radial molecular thickness as function of distance from the spacecraft mid-plane (normal to axis at its midpoint).

For this purpose we computed the ring-symmetric exhaust flow field, using a semi-inverse marching characteristics scheme [7] . The marching was in the radial direction, starting with uniform flow at the nozzle exit; the computation was carried on until it became evident that even at a distance of 20 (m) from the mid-plane, the radial molecular thickness was well over 40. The entire spacecraft was thus shielded from any ambient scattering at (or near) normal incidence. This shielding effect has two significant implications which we discuss briefly below.

(a) It is present only during stationary exhaust flow. At startup and shutdown phases, ambient scattering may be substantial even at normal incidence.

(b) During the stationary phase, ambient scattering is substantial only at attitude angles that enable ambient molecules to reach the vicinity of the plume by traveling through "molecularly thin" cavitation regions that flank the plume. We thus anticipate a decisive dependence of ambient scattering on attitude variations, whenever those variations steer the spacecraft into or out of a shielded posture.

As a first attempt at a quantitative estimate of ambient scattering flux, we have formulated a simple first-collision model of this effect. In the sequel we present an outline of the model, along with some sample results evaluated for an HF laser configuration identical to that considered for the shielding effect mentioned above.

The basic idea is the following. Ambient molecules entering an exhaust plume, require several collisions to become fully "accommodated" with the main flow (i.e., to be entrained by the main flow at the prevailing flow velocity and temperature). One may reasonably approximate this process by considering just one collision - the first.

With the help of some additional assumptions, we were able to derive a closed form expression for the flux of exhaust molecules that arrive at the spacecraft following a first collision with an ambient molecule. The main assumptions of this model are :

(1) FIRST COLLISIONS: Only first collisions for either ambient or exhaust molecules are considered. Hard-spheres elastic collisions are assumed. Upon a second collision of either an ambient or an exhaust molecule, it is considered "lost" (i.e., it joins the main flow). Collisions of ambient molecules with spacecraft surfaces are ignored. Ambient molecules are assumed to traverse cavitation regions collisionlessly.

(2) COLD FLOW: The oncoming ambient air flow is deemed "cold"; i.e., all molecules move at the uniform orbital velocity. The same "cold" assumption is applied to the exhaust flow, since most ambient scattering takes place at plume regions of very high Mach numbers (well over 10, in the present case).

(3) CRW Flow Field: ring-symmetric CRW flow field is determined from the power-law approximation described in Ch. 2 above. This approximation approaches Prandtl-Meyer flow at points whose distance from the nozzle lip is much smaller than the spacecraft radius.

Based on these assumptions, ambient scattering is represented as a source term for side-scattered exhaust molecules, distributed throughout the lip-centered rarefaction fan. The total flux arriving at a specified point on the cylindrical spacecraft is readily computed by integrating numerically that source distribution over the entire ring-fan.

The highlights of the spatial integration scheme (Fig. 3-1) are as follows. The limiting characteristic surface ($M = \infty$) of the ring-symmetric CRW is divided into surface elements formed by dividing the surface into a set of ring-strips which are subdivided in the circumferential (azimuthal) direction (φ) into surface elements. The line-of-sight ($\vec{\Omega}$) from the "target point" on the spacecraft to the center of each surface element is extended into the ring-symmetric CRW, and flux integration using the first-collision source term with appropriate weight factors is performed along this line until convergence is attained. Contributions from each surface element are summed, taking care to disregard portions of the ring-symmetric CRW that are shadowed by the cylindrical spacecraft (either the line-of-sight or the trajectory of oncoming ambient molecules may be shadowed). Some further details of the flux integration scheme and hard-spheres collisions are provided in Section 3.2 below.

3.2 Flux Integration Scheme

The description of the first collision model is hereby supplemented with an outline of the expressions used in the flux integration and their derivation. The integration scheme for flux arriving at point X_s on the spacecraft is depicted in Fig. 3-1. Note that only the plane of incidence is shown in Fig. 3-1; at other azimuth angles the geometry is not co-planar, so 3-D geometrical expressions are used to get the coordinates (ψ,φ and radial distancete $(y^2 + z^2)^{1/2}$) from $\vec{\Omega}$ and S; the derivation of these geometrical relations is straightforward, so that we omit these details in the present report. The total number flux $Q_i(X_s)$ of i exhaust molecules arriving at point X_s is given by the following expression :

$$Q_i(X_s) = \int d^3\vec{\Omega}\, \cos\alpha_s \sum_k \int_0^{\infty} dS\, \sigma_{ik}\, h_i\, N(S)\, h_k\, N_A\, \left|\vec{U}(S) - \vec{U}_A\right|\, \exp[-\eta_k(S)]\, P_{ik}(S, -\vec{\Omega})\, \exp[-\eta_{ik}(S)]$$

$$\eta_k(S) = \sum_i \int_0^{t(S)} dt'\, \sigma_{ik}\, h_i\, N(t')\, \left|\vec{U}(t') - \vec{U}_A\right| / \left|\vec{U}_A\right| \qquad (3\text{-}1)$$

$$\eta_{ik}(S) = \sum_j \int_0^{S} dS'\, \sigma_{ij}\, h_j\, N(S')\, \left|\vec{U}_{ik}(S) - \vec{U}(S')\right| / \left|\vec{U}_{ik}(S)\right|$$

$()_i$ $()_j$ – Exhaust species $()_k$ – Ambient species

These expressions are interpreted as follows. The collision depicted in Fig. 3-1 is between exhaust molecule m_i and ambient molecule m_k. The exhaust molar fractions h_i and ambient molar fractions h_k are assumed uniformly constant, and so are the ambient velocity $\vec{U}_A$ and number density N_A. The exhaust velocity $\vec{U}(S)$ and number density $N(S)$ are function of the location in the flow field defined by $\vec{\Omega}$ and S. These flow variables are computed by first evaluating the coordinates of point $\vec{\Omega}.S$ (Fig. 3-1) in the ring-symmetric CRW from the 3-D geometry, and then employing the power-law approximation outlined in Ch. 2 above, to get all flow variables for a ring-symmetric CRW. In this computation we exploit the fact that characteristic lines fanning out from the nozzle lip are nearly straight lines at the low pressure side of the ring-symmetric CRW.

The $\vec{\Omega}$ integration is performed numerically according to the scheme outlined in Section 3.1 above, as a summation over elements of solid angle ($\Delta^3\vec{\Omega}$) subtended by area elements on the limiting characteristic cone ($\psi = \psi_f$).

The S integration is considerably more complex. The integrand for this integration is derived as follows. Denote by L the line-of-sight distance between point X_s and fan point $\vec{\Omega}.S$. A volume element at the fan point is given by $\Delta v = L^2\, \Delta S\, \Delta^3\vec{\Omega}$. The number of ik pair collisions in Δv per unit time is $\sigma_{ik}\, h_i\, N(S)\, h_k\, N_A\, \left|\vec{U}(S) - \vec{U}_A\right|\, \exp[-\eta_k(S)]\, \Delta v$, where $\eta_k(S)$ denotes the expected number of collisions of ambient molecule k with any exhaust molecule, between its point of entry into the plume and point $\vec{\Omega}.S$. We now multiply this term by $\exp[-\eta_{ik}(S)]$ which is the probability that exhaust molecule i scattered by ambient molecule k would travel from point $\vec{\Omega}.S$ to point X_s collisionlessly, where $\eta_{ik}(S)$ is the expected number of collisions for this path segment. (Note that in Eq. (3-1) the summation in the expression for $\eta_{ik}(S)$ is over all exhaust species j).

The final step in constructing the integrand for the S integration involves the post-collision directional distribution function $P_{ik}(S, -\vec{\Omega})$, whose derivation will be given in the sequel. We multiply the integrand by $P_{ik}(S, -\vec{\Omega})\, \Delta^3\vec{\Omega}_c$ which is the fraction of i exhaust molecules scattered by k ambient molecules into a solid angle element $\Delta^3\vec{\Omega}_c$ about the unit vector $-\vec{\Omega}$. Considering the flux arriving at a surface area element ΔA_s around point X_s, the solid angle element subtended by ΔA_s is $\Delta^3\vec{\Omega}_c = \Delta A_s \cos\alpha_s / L^2$. Eq. (3-1) for $Q_i(X_s)$ now follows upon dividing the resulting expression by ΔA_s, thus referring the arriving flux to a unit area at the point of arrival X_s.

Numerically, the S integration was performed using the classical Runge-Kutta scheme (fourth order). The integration for $\eta_{ik}(S)$ and $\eta_k(S)$ has to be repeated at each point S. We found reasonable convergence with 4 points in the $\eta_k(S)$ integration and 6 points in the azimuth integration. The S integration was terminated when convergence was attained (this is the meaning of the upper limit ∞ in the S integral in Eq. (3-1)). The summation over new strips on the limiting cone ($\psi = \psi_f$) was also terminated upon convergence. The CPU time consumed per target point was about 100 (sec) on IBM 3033 mainframe.

We now take up the derivation of an expression for the post-collision directional distribution function $P_{ik}(S, -\vec{\Omega})$, which we denote hereafter as $P(-\vec{\Omega})$. We adopt the pair-collision notation presented in Fig. 3-2 for the hard-sphere collision analysis.

As a consequence of conservation of momentum and energy (elastic collisions), the center-of-mass velocity $\vec{C}_m$ and the magnitude of the relative velocity $\vec{C}_r$ are unchanged by the collision [8] . The post-collision velocities are given by :

$$\vec{C}_1^* = \vec{C}_m + \mu_2\vec{C}_r^* \qquad \vec{C}_2^* = \vec{C}_m - \mu_1\vec{C}_r^*$$

$$\vec{C}_r = \vec{C}_1 - \vec{C}_2 \qquad \vec{C}_r^* = \vec{C}_1^* - \vec{C}_2^* \qquad (3\text{-}2)$$

$$\mu_1 = m_1/(m_1 + m_2) \qquad \mu_2 = m_2/(m_1 + m_2)$$

$$\vec{C}_m = \mu_1\vec{C}_1 + \mu_2\vec{C}_2 \qquad |\vec{C}_r^*| = |\vec{C}_r|$$

The only free parameter in the expressions for post-collision velocities is the orientation of the post-collision relative velocity $\vec{C}_r^*$. This orientation is uniformly likely to be in any direction in space when hard-spheres collision is assumed [8] , as represented by the spherical scattering envelope in

Fig. 3-3. The probability of obtaining $\vec{C}_1^*$ in solid angle element $\Delta^3\vec{\Omega}$ about $-\vec{\Omega}$ (Fig. 3-3) is given by :

$$P(-\vec{\Omega}) = \left(1/4\pi\left|\mu_2\vec{C}_r\right|^2\right)\left(\Delta A/\Delta^3\vec{\Omega}\right) = \left(1/4\pi\left|\cos\delta\right|\right)\left(\left|\vec{C}_1^*\right|^2/\left|\mu_2\vec{C}_r\right|^2\right) \qquad (3\text{-}3)$$

where ΔA is an area element on the scattering envelope, whose projection on a plane normal to $\vec{\Omega}$ is $\Delta A\left|\cos\delta\right|$. We note that the origin of $\vec{C}_m$ in Fig. 3-3 is external to the scattering envelope, resulting in two possible scattering elements on the sphere. In all the cases that we computed, however (see Ch. 4 below), that point was found to be always internal, so that there was only a single scattering solution with post-collision velocity $\vec{U}_{ik}(S)$ pointing at the spacecraft for any ik pair collision.

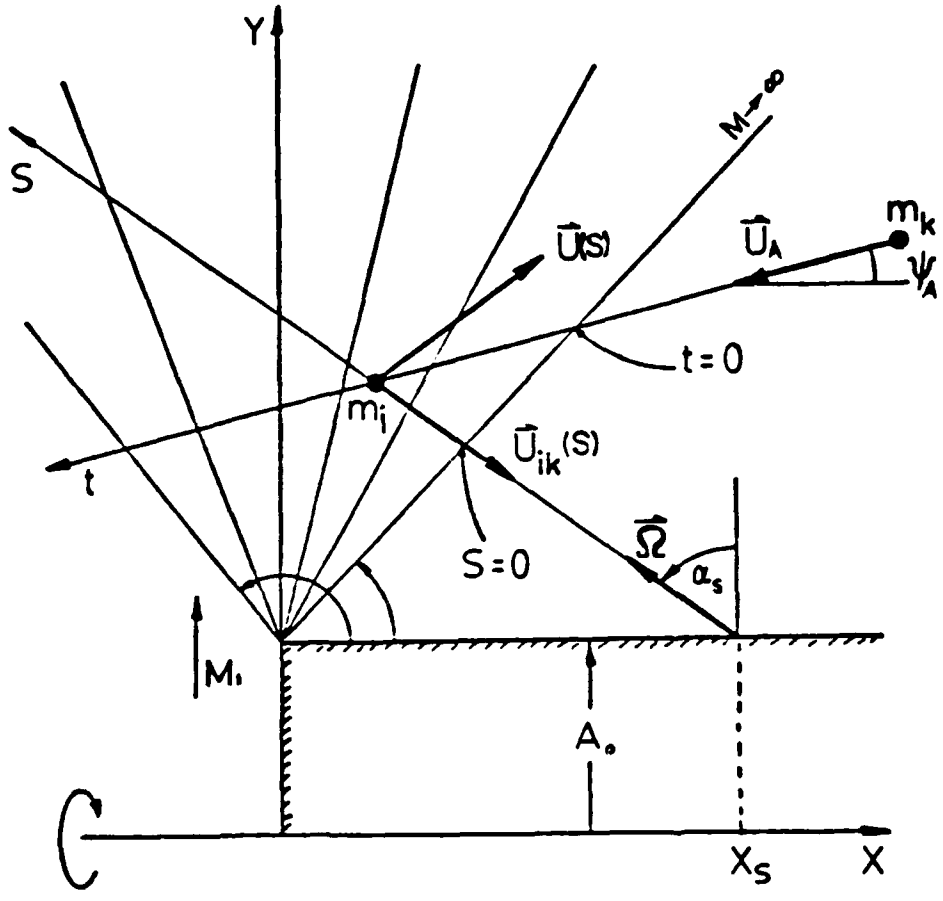

Figure 3-1. Incidence-Plane Description of Flux Integration Scheme.

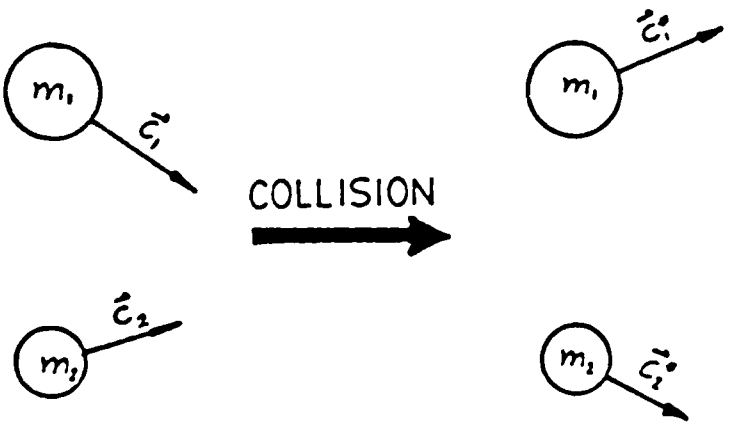

Figure 3-2. Hard-Spheres Collision Notation.

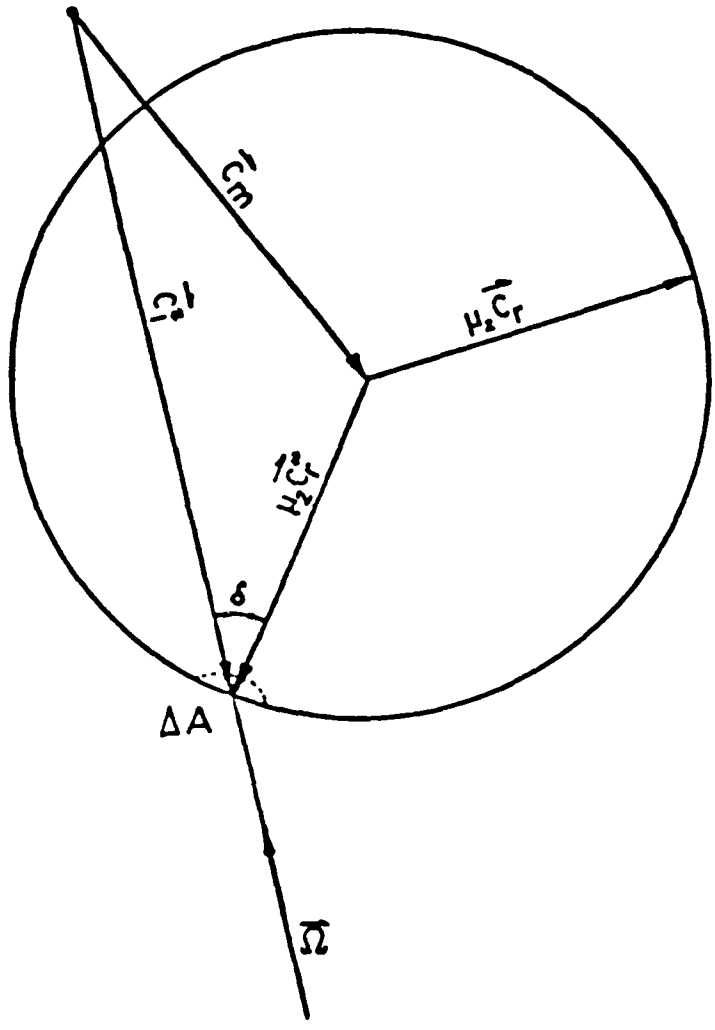

Figure 3-3. Scattering Envelope for Hard-Spheres Collision.

4. RESULTS AND DISCUSSION

We performed several computations of return flux generated by ambient scattering, aimed at demonstrating the expected flux level and its variation with spacecraft target point and orbital attitude angles. In all these computations we assumed that the exhaust flow is as in the typical HF/DF laser case (Table 4-1 below), and that the ambient density and velocity are $N_A = 1 \times 10^{16}$ (molecules/m^3) and $U_A = 8$ (km/sec). As an approximation we further assumed that the sole ambient species is molecular nitrogen (molecular weight $W_A = 28$) and that all binary collision cross-sections are uniformly given by $\sigma = \pi D^2$, where D is the molecular diameter (Table 4-1). In each computation we evaluated the combined HF + DF flux by assuming that the molar fraction of DF is zero and the molar fraction of HF is the combined value for both species (Table 4-1) : .091 + .135 = .226 . This is justified by the relatively small difference in molecular weight (just 5%) between these two species.

Three sets of flux computation were performed as follows :

(a) Incidence-plane ($\varphi_A = 0$) target points at various distances from the nozzle lip ($X_s = .1$ to $X_s = 10$ (m)), and at constant incidence angle ($\psi_A = 20°$). The results are shown in Fig. 4-1. We observe that the flux is fairly insensitive to X_s. Also shown in Fig. 4-1 are flux computations where the ring-symmetric CRW flow is approximated as a planar CRW (Prandtl-Meyer flow), rather than the power-law as in Eq. (2-1) above. The planar case exhibits a somewhat higher flux, particularly at large X_s.

(b) Incidence-plane ($\varphi_A = 0$) target points at $X_s = 1$ (m) and at various incidence angles ($\psi_A = 0$ to $\psi_A = 40°$). A polar representation of the results is given in Fig. 4-2. Note the sharp decrease in flux as the incidence angle ψ_A approaches the plume limiting angle $\psi_f = 41°$.

(c) Azimuth angle variation ($\varphi_A = 0$ to $\varphi_A = 180°$) at a constant location ($X_s = 1$ (m)) and at a constant angle of incidence ($\psi_A = 20°$). A polar representation of the results is given in Fig. 4-3. Observe that flux becomes sensitive to azimuth angle φ_A only past $\varphi_A = 90°$, where shadowing by the cylindrical spacecraft becomes increasingly dominant.

In addition to return flux we also computed the rms velocity of the arriving molecules. For the target points in group (a), the rms velocity varied between 6000 and 6600 (m sec) (the higher velocity at smaller X_s), which corresponds to a kinetic energy of about 4 (ev) per molecule (HF).

The maximum return flux arriving at the spacecraft is about 0.15×10^{19} (molecules/m^2sec), which corresponds to a surface deposition rate of about 300 monolayers (HF + DF) per hour. This level of contaminating flux may seem to be not outright negligible; however, since return flux is proportional to ambient density, it will be scaled down considerably at higher altitudes (and lower ambient densities).

We observe that the maximum return flux constitutes a fraction of about 2% of the incident ambient flux. This return flux ratio is roughly maintained at almost all target points and attitude angles in groups (a), (b) and (c). The only exceptions are incidence angles near the limiting cone ($\psi = \psi_f$) or at azimuth angles $\varphi_A > 125°$ where shadowing becomes dominant. This observation is interpreted as follows.

Consider the total solid angle subtended by the limiting cone (considered to be infinitely extended in the axial direction) as viewed from a target point (for all lines-of-sight $\vec{\Omega}$ pointing outward of the cylindrical spacecraft surface). It is independent of target location due to the "self-similar" geometry. During each flux computation, we also evaluated the total solid angle subtended by that segment of the cone over which the flux integration was actually performed (see Section 3.2). It was found out that for all but the "shadowed" cases ($\varphi_A > 125°$), this solid angle constituted a fraction of $86 \pm 1\%$ of the solid angle subtended by the infinite cone. We interpret this result as a hint that geometrical "view factors" arising in the course of the flux integration, are not the dominant factor in determining the 2% level of flux ratio. What then are the dominant factors?

For a possible explanation we turn to the flux integration scheme presented in Section 3.2. The flux ratio is obtained upon dividing the integrand in Eq. (3-1) by $N_A U_A$ and setting $h_k = 1$ (since we assume a single species air). The major factors in the flux ratio integrand appear to be the no-collision probabilities $\exp[-\eta_{ik}(S)]$ and $\exp[-\eta_k(S)]$, and the post-collision directional distribution function $P_{ik}(S, -\vec{\Omega})$. The flux-averaged values of these functions in the group (a) computations were found to be as follows : $P_{ik}(S, -\vec{\Omega}) = .09$ to $.10$. $\eta_{ik}(S) = .42$ to $.54$ and $\eta_k(S) = .35$ to $.47$. The flux-averaged Mach number for group (a) points exhibited a much larger variation : between 30 and 80, with the higher Mach numbers obtained at further target points.

These results are interpreted as follows. The ambient no-collision probability $\exp[-\eta_k(S)]$ is sufficiently close to unity, so that in an order-of-magnitude analysis such as the present one, we may disregard this factor. If the velocity ratio in the $\eta_{ik}(S)$ integral of Eq. (3-1) is assumed to be unity (its average value for group (a) points is about 1.4), then the differential in the flux S integration becomes

$\sigma N(S)dS = d\eta_{ik}(S)$. This implies that the flux S integration results in some average value of the only remaining factor : $h_i P_{ik}(S, -\vec{\Omega})$. Since the $\vec{\Omega}$ integration introduces a factor of order unity, the order-of-magnitude estimate for the arriving-to-incident flux ratio is $[h_i P_{ik}(S, -\vec{\Omega})]_{av}$. The value of this estimate is $[h_i P_{ik}(S, -\vec{\Omega})]_{av} = .226 \times .09 \approx .02$, which is about equal to the actual flux ratio for target points in group (a).

When an exhaust flow and orbital parameters (velocity and attitude) are specified, $P_{ik}(S, -\vec{\Omega})$ depends on the choice of molecular collision model (we chose hard spheres), while h_i is uniformly constant. The foregoing reasoning thus establishes the collision model as a significant factor in determining ambient scattering flux levels, to the extent that $P_{ik}(S, -\vec{\Omega})$ is sensitive to the choice of model.

Table 4-1. Typical Operating Conditions of HF/DF Laser Exhaust

Mole fractions	[H] = .091 [HF] = .091 [H_2] = .104 [DF] = .135 [He] = .579	
Average molecular weight	7.14	
Specific heats ratio	1.54	
Stagnation temperature and density	1400 (K) .0075 (kg. m^3)	
Exit Mach number	4.0	
Molecular diameter (hard spheres)	2.5×10^{-10} (m)	
Spacecraft diameter	5.0 (m)	
Nozzle aperture	1.0 (m)	

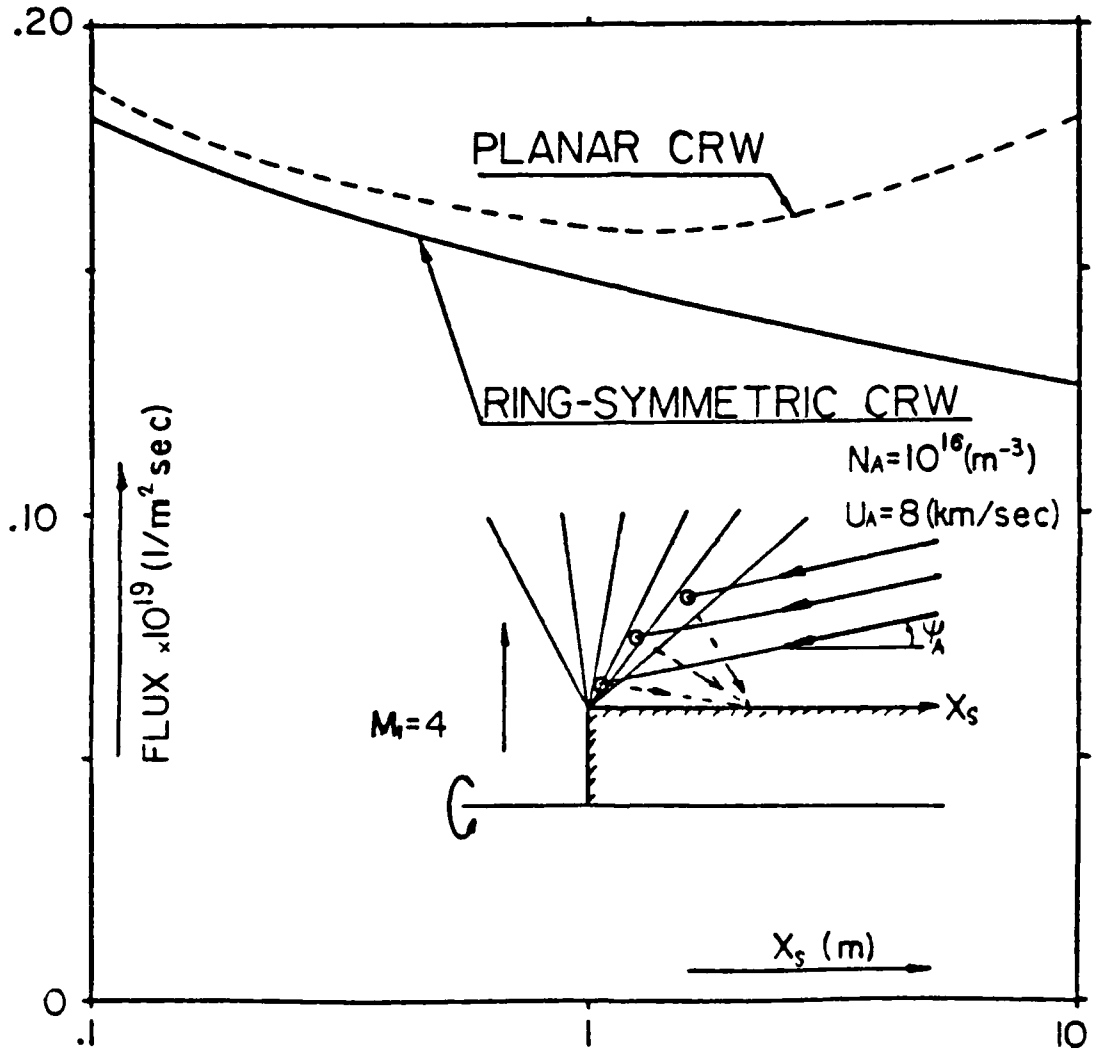

Figure 4-1. Variation of Return Flux with Target Point (X_s). Target Point at Incidence-Plane ($\varphi_A = 0$) and Constant Incidence-Angle ($\psi_A = 20°$).

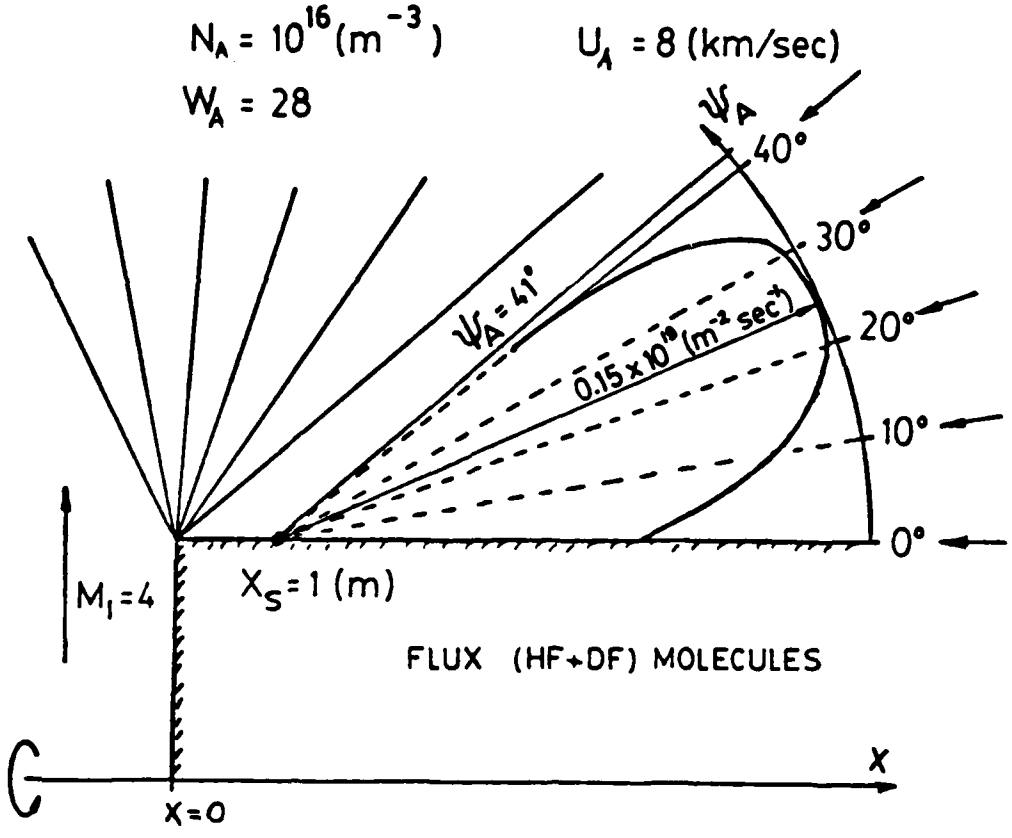

Figure 4-2. Variation of Return Flux with Ambient Incidence Angle (ψ_A). Fixed Target Point ($X_s = 1$ m) Located at Incidence-Plane ($\varphi_A = 0$).

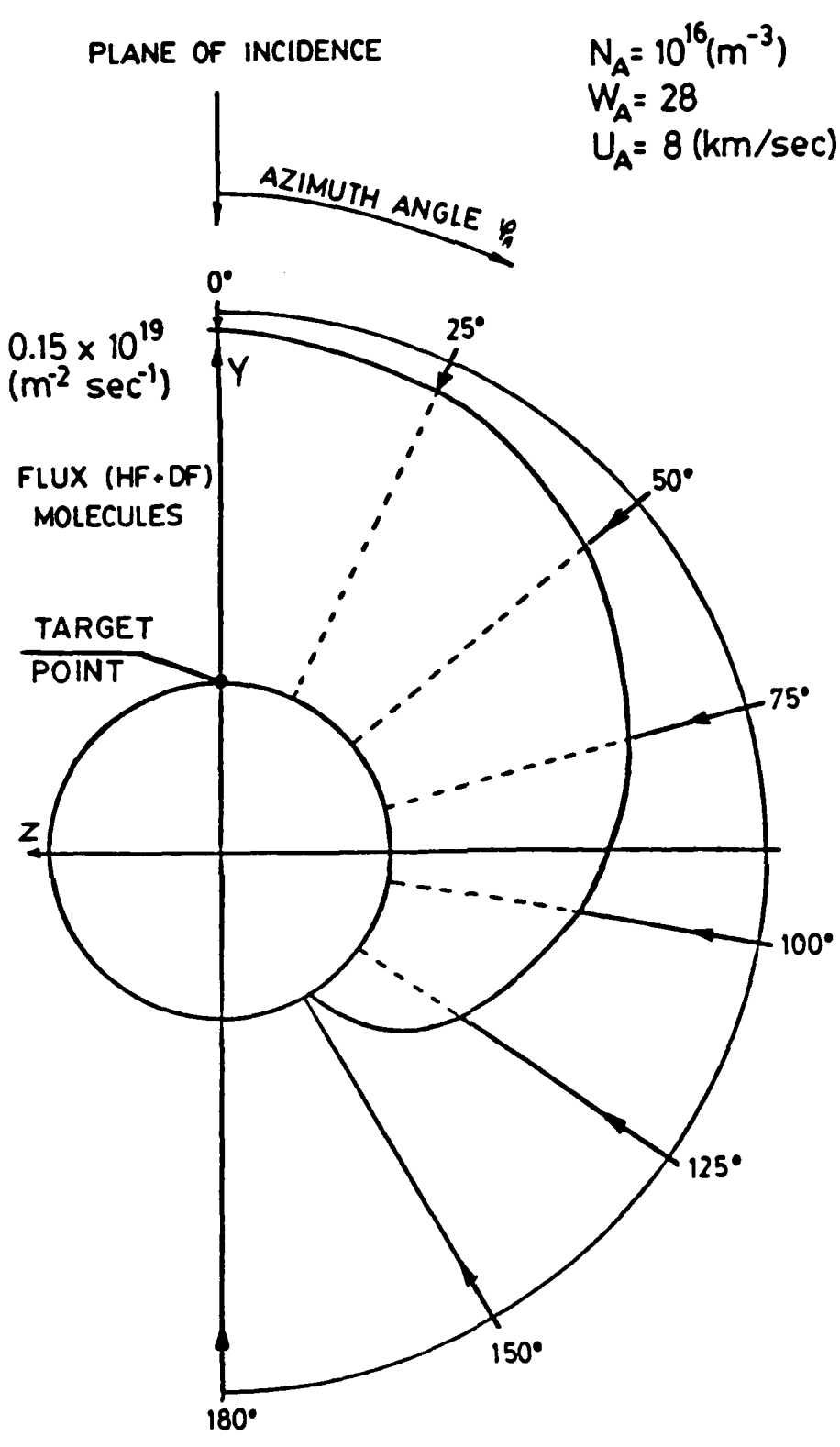

Figure 4-3. Variation of Return Flux with Ambient Azimuth Angle (φ_A). Fixed Target Point ($X_s = 1$ m) and Ambient Incidence Angle ($\psi_A = 20°$).

5. SPACECRAFT CHARGING

Spacecraft charging is a major concern to spacecraft designers, particularly for missions in GEO and to a lesser extent also in LEO. The exhaust plume of an HF/DF laser operating in the ionosphere (300 to 1000 km altitude) may modify significantly the pre-firing charging pattern of the spacecraft. Two classes of effects may lead to charging modification; they are:

(a) The exhaust contains large concentrations of HF and DF molecules which are highly electronegative. They may be readily ionized by environmental electrons and change the existing spacecraft charging pattern.

(b) When the spacecraft is oriented obliquely relative to its orbital velocity and the ambient plasma impinges at the plume boundary, the plume will cast a "shadow" on the downstream side, leading to a very dissimilar charging fluxes on the upstream and downstream halves of the spacecraft.

The knowledge gained in analyzing the ambient scattering effect can be applied to the assessment of the effects of ionospheric plasma on spacecraft charging. We first consider the upstream side of the spacecraft as mentioned in (a) above.

We contend that the exhaust-plasma interaction will not drastically alter the charging pattern of the upstream half. This assessment is established as follows. Consider the fact that ionospheric plasma has a particle number density no higher than 10^{12} (m^{-3}) and energy per particle of at most 1 (ev) (excluding the auroral plasma of polar zones or events of sun storms, where the energy per particle is much higher). Significantly, the Debye length at the highest plasma density is very small : only about 10^{-3} (m); the largest Debye length in the ionosphere is 10^{-1} (m) [9] . Ion thermal velocity is typically lower than orbital velocity, but electron velocity is considerably higher than orbital velocity (at 1 ev the electron velocity is about $U_e = 6 \times 10^5$ m sec). Hence, ions would typically impinge at the plume as a uniform ion beam with the orbital velocity (like ambient molecules), while electrons are expected to impinge at the plume with their random-oriented thermal velocity.

In view of the results of ambient scattering analysis (Ch. 3 and 4 above), and since ions are subject to similar collision process with exhaust molecules as neutrals, ions will be stopped at the plume fringes much like ambient molecules. By virtue of the small Debye length (typically much smaller than the stopping distance), electrons would not penetrate any further than ions, regardless of their

collision cross-section with exhaust molecules. The familiar plasma sheath that forms at a solid surface, is hence replaced at the plume/plasma boundary by a typically neutral layer whose thickness is of the order of an ion/neutral mean free path, but much larger than the Debye length. Only at the upper altitude range of the ionosphere does the Debye length become comparable to a plume boundary mean free path (about .1 m), but there plasma density and flux are several orders of magnitude lower and charging modification is not likely to be significant at the relatively short firing duration of about 5 minutes.

Elastically scattered ions can be deflected towards the spacecraft as a result of elastic collisions with exhaust molecules, much like neutrals. Referring to our analysis of the return-to-ambient flux ratio (Ch. 4 above), it is clear that the relevant ratio here will be about $1/4\pi$, i.e., of the order of 10% (this is due primarily to the role played by the elastic directional distribution function – see Ch. 4). A change in the plasma-to-surface current of that order is hence possible, but unlikely to affect spacecraft design or operation significantly. The reason is that a design capable of smoothing away the inhomogeneous charge flux at oblique attitudes, will not be sensitive to a change in flux pattern of the order of 10% (in other words, potential differences may be amplified by 10%, which is hardly likely in a sound design to bring about arcing or other threshold phenomena).

Another effect which may potentially be significant in the upstream half is generation of electronegative species (HF^-, DF^-) by plasma electrons impinging at the plume. In the sequel, we examine the magnitude of this effect, concluding that it is negligible.

This estimate is best done by considering $\dot{N}^-$, which is the rate of production of HF^- and DF^- per unit volume, at a typical point in the exhaust where local Mach number is $M = 30$ (this is typically the lowest average Mach number for the plume region where ambient scattering takes place – see Ch. 4 above). Since energy is released by the electronegative ion formation, the reaction involves a third body as follows :

$$HF + e^- + M \rightarrow HF^- + M \qquad (5\text{-}1)$$

where M is the third body molecule. We assume a simplified classical kinetic model for this reaction, as follows. The pair HF M collide with a frequency proportional to the local number density and HF molar fraction, and to the average relative velocity. An electronegative ion formation can occur only if an electron collides with the pair during their collision, which lasts $t_c = D/\bar{C}_r$, where $\bar{C}_r$ is the average relative pair velocity. Based on this classical model, and assuming the same cross-section for

electronegative ion formation as for elastic HF/M collisions, the volume rate of electronegative ion generation is given by :

$$\dot{N}^- = (\pi D^3 N)\, Nh\, (\pi D^2 U_e N_e) \qquad (5\text{-}2)$$

where $(\pi D^3 N)$ is the probability that a certain HF or DF molecule will be in contact (**D** being molecular hard-sphere diameter) with any other exhaust molecule (whose number density is **N**). When $(\pi D^3 N)$ is multiplied by **hN**, where **h** is the HF + DF molar fraction (Table 4-1), the combined term reads as the number of colliding HF/M pairs per unit volume. Assuming the electronegative formation cross-section is also πD^2, the factor $\pi D^2 U_e N_e$ where U_e and N_e are electron velocity and number density, renders the expression for electronegative generation rate per unit volume. We note that $\bar{C}_r$ cancels out in deriving Eq. (5-2), so that $\dot{N}^-$ does not depend on temperature. This supports the use of the kinetic approximation in regions of continuum breakdown (plume fringes are such regions).

How is the relative magnitude of $\dot{N}^-$ decided? To do that we multiply $\dot{N}^-$ by $\lambda = 1/\pi D^2 N$, which is the mean free path for a fast moving particle that penetrates the plume. This expression is justified by the fact that most incident particles do collide within a distance of order λ, and when the particles are plasma ions, electrons will adhere to ion spatial distribution by virtue of the small Debye length (smaller than λ). Thus, $\lambda\dot{N}^-$ is the rate of electronegative ion generation per unit area of plume boundary. The ratio β^- between this rate and the incident electron flux is :

$$\beta^- = \lambda\dot{N}^-/N_e U_e = (\pi D^3 N)h = 2.2 \times 10^{-10} \qquad (5\text{-}3)$$

where $N = 2 \times 10^{19}$ (m^{-3}) which corresponds to Mach number **M** = 30 in the typical case (Table 4-1). The fraction of electron flux captured by HF and DF exhaust molecules to form electronegative ions is so small (due to the pair-formation term $(\pi D^3 N)$), that it cannot appreciably alter the charging flux distribution at the spacecraft surface.

Another possible effect is the recoil of HF^- or DF^- that occurs due to energy released in the electronegative formation reaction. The recoiling species might conceivably reach the surface and contaminate it. The magnitude of the recoil flux is certainly no larger than $\beta^- U_e N_e = 1300$ (m^{-2} sec^{-1}), where we assume the worst case flux : $N_e = 10^{12}$, $U_e = 6 \times 10^5$ (m sec) which corresponds to about 1 ev energy per electron. This flux level is about 3×10^{-13} monolayers of HF^- and DF^- per hour, so that its contribution to surface contamination is utterly negligible.

The second kind of charging effects (item (b) above) is due to the fact that the exhaust plume is impenetrable to ambient plasma (within a range of sufficiently small distance from the spacecraft, so that no extensive diluting of the plume has taken place). The downstream half of the spacecraft in oblique attitude will be in the "shadow" with respect to incident plasma. As a first approximation we may assume zero plasma flux at the shadowed surface. More accurately, this portion of the spacecraft will be subject to a plasma wake flow. However, it is quite difficult to determine the charging phenomena that take place in such a wake, as indicated by a recent work on solar sails in LEO [9] . Thus, a zero flux at the downstream half seems a practical design assumption.

Can adverse charging effects occur as a result of shadowing the downstream half? This question can be discussed only qualitatively. The reason is that a quantitative analysis requires a lumped-circuit model of the spacecraft external surface [10] . Since such a concrete design is not available, we can only discuss this question qualitatively. Obviously, assuming zero flux to the downstream half during the envisioned 5 minutes of laser firing duration, and requiring that no appreciable voltages between the two halves will evolve, leads to the stipulation that the equivalent-circuit Capacitance × Resistance should be much smaller than the firing duration.

6. CONCLUDING REMARKS

Our major quantitative conclusion is that for the relatively high ambient density assumed ($N_A = 1 \times 10^{16}$ molecules/m^3 which represents Sunspot Maximum at about 200 km) and for the typical HF/DF laser exhaust (Table 4-1), the HF + DF flux backscattered by ambient molecules is several hundred monolayers per hour. This flux level may seem as not outright negligible. However, since ambient scattering flux is proportional to ambient density, it will be scaled down considerably at the lower ambient densities of higher orbital altitudes.

The operational scenario for HF/DF laser envisions 4 or 5 minutes total operating time; hence the contamination by ambient scattering may not be serious due to short operating time.

The effects of laser exhaust plume on spacecraft charging in the ionosphere were examined. It was concluded that the rate of electronegative (HF^- and DF^-) production by impinging electrons was negligible; the low rate is a consequence of the assumption that a third body is required to interact simultaneously with the HF/e or DF/e pair. No significant modification of charging pattern is anticipated. However, at oblique orbital attitudes, the downstream half of the spacecraft will be shadowed from the oncoming ambient plasma. This fact has to be reckoned with in designing a ring-symmetric laser spacecraft.

The emphasis in this work was on the method rather than on results. The first-collision model was demonstrated to be simple to implement in a code. It is considerably simpler than the more general and potentially more accurate Monte Carlo methods commonly used for simulating rarefied flows [8]. We found out that the molecular collision model was all important in determining the return flux level, which is hardly surprising for scattering by single collision. For the same reason, the collision model would also be dominant in a Monte Carlo simulation of the ambient scattering process.

If and when a mathematical accuracy of the first-collision approximation is established for hard-spheres, it might be possible to determine a realistic collision model by comparing computed results with measurements.

This accuracy may be established in either of two ways. One way is by comparison with accurate Monte Carlo computations (using hard-spheres collision model). The other is to seek an estimate of the error incurred by considering just first collisions and ignoring all subsequent ones. This might be achieved by accounting for second collisions in an extended first-collision model, provided a simplified

scheme that will obviate the need for increase in the dimensions of the numerical flux integration can be devised. We are currently considering such second-collision approaches.

7. REFERENCES

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APPENDIX A. DESCRIPTION OF AMB CODE

A.1 Description of Subroutines

We provide a list of the subroutines in the ambient flux integration code AMB for ring-symmetric cylidrical spacecrafts. Each subroutine is briefly described. Statements are identified by the FORTRAN statement number (columns 1 through 5).

MAIN PROGRAM The 300 loop is intended to enable several (NCASE) reruns with various data in each, all in a single run. Upon calling INIDAT1, parameters depending on data defined in INIDAT are re-computed. The 200 loop is over various XSV(NX) target points. In the 20 loop the flux integration begins : FLUXC is for particle flux and FLXU2C is for the rms of velocity of return flux molecules. All the MAX suffixed parameters denote values at which the integrand had the largest value.

The actual flux integration commences at statement 1 for the summation over strips of constant RF. This summation is terminated when convergence is attained (to within EPSR). The inner loop 2 is over azimuth angle PHI. Note that the target points are generally not in the plane of incidence (PHIA.NE.0), so that no symmetry can be assumed in the PHI integration, and it is performed twice in order to cover the entire range in PHI (IPAR=1 for PHI.GT.0, IPAR=2 for PHI.LT.0). The flux integration along the line-of-sight is done by calling FLUX.

INIDAT Initialization of data. There is no input file for this code. INIDAT1 is for parameters computed from the data defined by calling INIDAT.

SOF Stopping routine, called when an error is detected. Here we also trigger a system error by computing DSQRT(-1), in order to obtain a calling sequence printout by the operating system.

FLUX This routine calls SUMT for flux integration of one exhaust species at a time.

LIMIT Here we compute the point of intersection of the line-of-sight with the leading characteristic cone. If they do not intersect, the distance of the intersecting point TLIM is set to a very large number.

SUMT This is the line-of-sight integration routine. Runge-Kutta scheme is used (even though an explicit integral is computed). Note that ETAK and ETAIK have to be computed through a separate integration at each point of the line-of-sight integration. The integration step DT (T=S/RF) is re-adjusted at each integration step. The integration is terminated when convergence is attained (to within EPST).

FETA Here the integrand for the line-of-sight flux integration is evaluated. The hard-spheres collision model is used to determine the post-collision directional distribution factor PIK. The flux-average of any variable (such as UIK**2 in present version), can be computed by summing it multiplied by flux and subsequently dividing by the total arriving flux (see loop 31 in MAIN PROGRAM).

PATHIK Here the molecular thickness ETAIK of the I exhaust species scattered by the K ambient molecule, is computed by integration along the line-of-sight.

FT This routine computes the integrand for the ETAIK integration in PATHIK.

PATHK The analog to PATHIK for K ambient molecule. TAU is the normalized integration variable along the trajectory of the penetrating ambient molecule. Note that SHADOW=.TRUE. when the trajectory passes through the cylindrical spacecraft surface before entering the fan.

FTAU Computes the integrand for the ETAK integration in PATHK.

FAN Computes the fan coordinates PSI, XP, YP for a point on the line-of-sight. It is used to determine the Mach number and flow angle from the power-law approximation (see MATCH).

FANT Computes the fan coordinates PSI, XP, YP for a point on the ambient molecule trajectory.

HMSET Prepares the vector HMV(I) which is the value of the H(M) integral at a set of Mach number values (equally spaced in inverse Mach number). This vector is used to compute H(M) for an arbitrary M (see HINTER), since this function is needed in the power-law approximation of flow in a ring-symmetric fan. Subsequent routines MFUNC, HINTER, MATCH and AREAF are all used to implement this approximation.

MFUNC Computes the integrand for the H(M) integration in HMSET.

HINTER Computes H(M) for a given M, from HMV(I) by linear interpolation. Note that the interpolation is done with inverse Mach number as the independent variable.

MATCH Here the approximation to the "inverse problem" of finding the Mach number at a single point in the ring-fan is implemented. An iteration scheme is used to determine the fan characteristic passing through the given point [2] .

AREAF Mach number is computed from value of area ratio function. Newton-Raphson iterations are used.

A.2 Listing of AMB code

```
C$OPTIONS LIST                                                          AMB0001
C  AMBIENT.  SCATTERING FROM A RING PLUME BY AMBIENT AIR.               AMB0002
      IMPLICIT REAL*8(A-H,O-Z)                                          AMB0003
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB0004
     1            G16,G17,G18,G19,G20                                   AMB0005
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,     AMB0006
     1          DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                      AMB0007
      COMMON /NPAR/NPHI,IPAR,NP,NR,NX,NXS,NS,NSPEC,NS1,NS2,NTAU0,NETA0, AMB0008
     1            NAMB,NCASE,ICASE,IFAN                               AMB0009
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,    AMB0010
     1            CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB0011
     2            PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0,  AMB0012
     3            DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)        AMB0013
      COMMON /EPSIL/EPSETA,EPST,EPSR                                    AMB0014
      COMMON /EXTREM/TEXT(5),ETAEXT(5),ETAKXT(5),PHIEXT(5),             AMB0015
     1               PSIEXT(5),EMEXT(5),FEXT(5),WEXT(5),                AMB0016
     2               TMAX(5),ETAKMX(5),ETAMAX(5),PSIMAX(5),             AMB0017
     3               EMMAX(5),FMAX(5),                                  AMB0018
     4               RFMAX(5),PHIFMX(5),PHIMAX(5),WMAX(5)               AMB0019
      COMMON /COUNTS/ICONTC,ICONTT,ICNTOT,ICNTMX,IQTOT(5),ISHAD(5)      AMB0020
      COMMON /SPEC/WAV,XC(5),WC(5),WRC(5),XNAME(5),QFC(5),QDC(5),       AMB0021
     1            QU2C(5),FLUXC(5),OMEGA(5),FLXU2C(5),URMSC(5)        AMB0022
      COMMON /AMBIEN/ENA,UA,PSIA,PHIA,HA(3),WA(3),                      AMB0023
     1               UAX,UAY,UAZ,AA,BA,CA,RA,XA,YA,ZA,SHADOW            AMB0024
      COMMON /POINT/XP,YP,XCOR,YCOR                                     AMB0025
      LOGICAL SHADOW                                                    AMB0026
      DIMENSION DSUMF(5),DSUMD(5),DSUMAX(5),DSUMU2(5)                   AMB0027
      NCASE=1                                                           AMB0028
      DO 300 ICASE=1,NCASE                                              AMB0029
      CALL INIDAT                                                       AMB0030
      GO TO (301,302,303,304,305,306,307,308,309,310,                   AMB0031
     1       311,312,313,314,315,316,317,318,319,320),ICASE             AMB0032
  301 CONTINUE                                                          AMB0033
      IFAN=1                                                            AMB0034
      NXS=3                                                             AMB0035
      XSI=0.1D0                                                         AMB0036
      GO TO 399                                                         AMB0037
  302 CONTINUE                                                          AMB0038
      PHIA=20.D0/DEG                                                    AMB0039
      GO TO 399                                                         AMB0040
  303 CONTINUE                                                          AMB0041
      PHIA=50.D0/DEG                                                    AMB0042
      GO TO 399                                                         AMB0043
  304 CONTINUE                                                          AMB0044
      PHIA=75.D0/DEG                                                    AMB0045
      GO TO 399                                                         AMB0046
  305 CONTINUE                                                          AMB0047
      PHIA=100.D0/DEG                                                   AMB0048
      GO TO 399                                                         AMB0049
  306 CONTINUE                                                          AMB0050
      PHIA=125.D0/DEG                                                   AMB0051
      GO TO 399                                                         AMB0052
  307 CONTINUE                                                          AMB0053
      PHIA=150.D0/DEG                                                   AMB0054
      GO TO 399                                                         AMB0055
  308 CONTINUE                                                          AMB0056
      PHIA=175.D0/DEG                                                   AMB0057
      GO TO 399                                                         AMB0058
  309 CONTINUE                                                          AMB0059
      GO TO 399                                                         AMB0060
  310 CONTINUE                                                          AMB0061
      GO TO 399                                                         AMB0062
  311 CONTINUE                                                          AMB0063
      GO TO 399                                                         AMB0064
  312 CONTINUE                                                          AMB0065
      GO TO 399                                                         AMB0066
  313 CONTINUE                                                          AMB0067
      GO TO 399                                                         AMB0068
  314 CONTINUE                                                          AMB0069
      GO TO 399                                                         AMB0070
  315 CONTINUE                                                          AMB0071
      GO TO 399                                                         AMB0072
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  316  CONTINUE                                                          AMB0073
       GO TO 399                                                         AMB0074
  317  CONTINUE                                                          AMB0075
       GO TO 399                                                         AMB0076
  318  CONTINUE                                                          AMB0077
       GO TO 399                                                         AMB0078
  319  CONTINUE                                                          AMB0079
       GO TO 399                                                         AMB0080
  320  CONTINUE                                                          AMB0081
       GO TO 399                                                         AMB0082
  399  CONTINUE                                                          AMB0083
       PRINT 101                                                         AMB0084
  101  FORMAT('1')                                                       AMB0085
C                                                                       AMB0086
       CALL INDAT1                                                       AMB0087
C                                                                       AMB0088
       DO 200 NX=1,NXS                                                   AMB0089
       XS=XSV(NX)                                                        AMB0090
C  (X0,Y0,Z0) IS THE POINT AT WHICH FLUX AND DENSITY ARE COMPUTED.       AMB0091
C  THE NORMAL TO THE SURFACE AT (X0,Y0,Z0) IS PARALLEL TO Y-AXIS.        AMB0092
       X0=XS                                                             AMB0093
       Y0=A0                                                             AMB0094
       Z0=0.                                                             AMB0095
       DO 20 NS=NS1,NS2                                                  AMB0096
       FLUXC(NS)=0.                                                      AMB0097
       FLXU2C(NS)=0.                                                     AMB0098
       OMEGA(NS)=0.                                                      AMB0099
       DSUMAX(NS)=0.                                                     AMB0100
       IQTOT(NS)=0                                                       AMB0101
       ISHAD(NS)=0.                                                      AMB0102
       TMAX(NS)=-1.D 44                                                  AMB0103
       ETAKMX(NS)=-1.D 44                                                AMB0104
       PHIMAX(NS)=-1.D 44                                                AMB0105
       PHIFMX(NS)=-1.D 44                                                AMB0106
       WMAX(NS)=-1.D 44                                                  AMB0107
       PSIMAX(NS)=-1.D 44                                                AMB0108
       ETAMAX(NS)=-1.D 44                                                AMB0109
       RFMAX(NS)=-1.D 44                                                 AMB0110
       EMMAX(NS)=-1.D 44                                                 AMB0111
       FMAX(NS)=-1.D 44                                                  AMB0112
   20  CONTINUE                                                          AMB0113
       RN=RMIN                                                           AMB0114
       APF=A0-0.5D0*DR0*SPSIF                                            AMB0115
       NR=0                                                              AMB0116
    1  NR=NR+1                                                           AMB0117
       DR=DR0                                                            AMB0118
       DR=DR0*(APF/A0)                                                   AMB0119
C      DR=DR0*(1.D0+0.4D0*DR0/XS)**NR                                    AMB0120
       RF=RN+DR/2.D0                                                     AMB0121
       APF=A0+RF*SPSIF                                                   AMB0122
       PHISOF=DACOS(A0/APF)                                              AMB0123
C      DPHI0=0.1D0                                                       AMB0124
C      NPHI=PHISOF/DPHI0+2                                               AMB0125
       DPHI=PHISOF/DBLE(NPHI)                                            AMB0126
       DO 21 NS=NS1,NS2                                                  AMB0127
       DSUMF(NS)=0.                                                      AMB0128
       DSUMU2(NS)=0.                                                     AMB0129
       DSUMD(NS)=0.                                                      AMB0130
   21  CONTINUE                                                          AMB0131
       DOMEGR=0.                                                         AMB0132
       DO 2 NP=1,NPHI                                                    AMB0133
       DO 2 IPAR=1,2                                                     AMB0134
       PHIF=(DBLE(NP)-0.5D0)*DPHI                                        AMB0135
       IF(IPAR.EQ.2) PHIF=-PHIF                                          AMB0136
C                                                                       AMB0137
       CALL FLUX                                                         AMB0138
C                                                                       AMB0139
       CROSS1=OMEGY                                                      AMB0140
       CROSS2=(SPSIF)*(-OMEGX)+(-CPSIF*CPHIF)*(-OMEGY)+                  AMB0141
     1        (-CPSIF*SPHIF)*(-OMEGZ)                                    AMB0142
       IF(CROSS1.LE.0.)                                                  AMB0143
     1CALL SOF('DIRECTION COSINE OF SURFACE NORMAL SHOULD BE POSITIVE')  AMB0144
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      IF(CROSS2.LE.0.)                                                  AMB0145
     1CALL SOF('NORMAL TO LIMITING CONE HAS NEGATIVE PROJECTION ON LINE-AMB0146
     1OF-SIGHT')                                                        AMB0147
      DOMEGA=CROSS2*DPHI*APF*DR/DIST**2                                 AMB0148
      DOMEGR=DOMEGR+DOMEGA                                              AMB0149
      DO 24 NS=NS1,NS2                                                  AMB0150
      DSUMF(NS)=DSUMF(NS)+DOMEGA*QFC(NS)*CROSS1                         AMB0151
      DSUMU2(NS)=DSUMU2(NS)+DOMEGA*QU2C(NS)*CROSS1                      AMB0152
      IF(DSUMAX(NS).GT.DOMEGA*QFC(NS)*CROSS1) GO TO 24                  AMB0153
      DSUMAX(NS)=DOMEGA*QFC(NS)*CROSS1                                  AMB0154
      TMAX(NS)=TEXT(NS)                                                 AMB0155
      ETAKMX(NS)=ETAKXT(NS)                                             AMB0156
      PHIMAX(NS)=PHIEXT(NS)*DEG                                         AMB0157
      PHIFMX(NS)=PHIF*DEG                                               AMB0158
      WMAX(NS)=WEXT(NS)*DEG                                             AMB0159
      PSIMAX(NS)=PSIEXT(NS)*DEG                                         AMB0160
      ETAMAX(NS)=ETAEXT(NS)                                             AMB0161
      RFMAX(NS)=RF                                                      AMB0162
      EMMAX(NS)=EMEXT(NS)                                               AMB0163
      FMAX(NS)=QFC(NS)*XC(NS)*Q0                                        AMB0164
   24 CONTINUE                                                          AMB0165
    2 CONTINUE                                                          AMB0166
      DO 26 NS=NS1,NS2                                                  AMB0167
      FLUXC(NS)=FLUXC(NS)+DSUMF(NS)                                     AMB0168
      FLXU2C(NS)=FLXU2C(NS)+DSUMU2(NS)                                  AMB0169
      OMEGA(NS)=OMEGA(NS)+DOMEGR                                        AMB0170
   26 CONTINUE                                                          AMB0171
      RN=RN+DR                                                          AMB0172
      IF(NR.LE.2) GO TO 1                                               AMB0173
      IF(NR.GT.99) GO TO 10                                             AMB0174
      DO 27 NS=NS1,NS2                                                  AMB0175
      IF(FLUXC(NS).EQ.0.) GO TO 27                                      AMB0176
      ERR=(DSUMF(NS)/FLUXC(NS))/DOMEGR                                  AMB0177
      IF(ERR.GT.EPSR) GO TO 28                                          AMB0178
   27 CONTINUE                                                          AMB0179
      GO TO 10                                                          AMB0180
   28 CONTINUE                                                          AMB0181
      GO TO 1                                                           AMB0182
   10 CONTINUE                                                          AMB0183
      DO 31 NS=NS1,NS2                                                  AMB0184
      FLUXC(NS)=XC(NS)*FLUXC(NS)*Q0                                     AMB0185
      OMEGA(NS)=OMEGA(NS)/(2.D0*PAI*DCOS(PSIF/2.D0)**2)                 AMB0186
      FLXU2C(NS)=XC(NS)*FLXU2C(NS)*Q0                                   AMB0187
      URMSC(NS)=0.                                                      AMB0188
      IF(FLUXC(NS).EQ.0.) GO TO 31                                      AMB0189
      URMSC(NS)=DSQRT(FLXU2C(NS)/FLUXC(NS))                             AMB0190
C  AVERAGE EM  (SEE FETA)                                               AMB0191
      URMSC(NS)=      FLXU2C(NS)/FLUXC(NS)                              AMB0192
   31 CONTINUE                                                          AMB0193
      PRINT 11,NX,NR,XS,RF,DR,PHISOF*DEG                                AMB0194
   11 FORMAT(///1X,'NX,NR,XS,RF.DR,PHISOF=',2I4,3D13.4,F8.4,            AMB0195
     1       3X,'FLUX AND EXTREMA VALUES, ALL SPECIES:'/)               AMB0196
      PRINT 12                                                          AMB0197
   12 FORMAT(/1X,' NAME ',' IQTOT',' ISHAD',                            AMB0198
     1          '   FMAX   ','   OMEGA','    TMAX',                   AMB0199
     2          '  ETAKMX','  ETAMAX','  PSIMAX',                     AMB0200
     3          '   EMMAX','   RFMAX',' PI-WMAX',                     AMB0201
     4          '   URMSC','  FLUXC   / LOG'/)                        AMB0202
      DO 14 NS=NS1,NS2                                                  AMB0203
      DLF=0.                                                            AMB0204
      IF(FLUXC(NS).NE.0)                                                AMB0205
     1DLF=DLOG10(FLUXC(NS))+100.D0+1.D-11                               AMB0206
      IDLF=DLF                                                          AMB0207
      DLF=DLF-DBLE(IDLF)                                                AMB0208
      PRINT 13,XNAME(NS),IQTOT(NS),ISHAD(NS),FMAX(NS),OMEGA(NS),        AMB0209
     1                          TMAX(NS),ETAKMX(NS),ETAMAX(NS),         AMB0210
     2                          PSIMAX(NS),EMMAX(NS),RFMAX(NS),         AMB0211
     3                          180.D0-WMAX(NS),URMSC(NS),              AMB0212
     4                          FLUXC(NS),DLF                           AMB0213
   13 FORMAT(1X,A6,2I6,D10.3,4F8.4,4F8.1,F8.2,D10.3,'/',F4.2)           AMB0214
   14 CONTINUE                                                          AMB0215
  200 CONTINUE                                                          AMB0216
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      PRINT 102                                                          AMB0217
  102 FORMAT(///1X,'END  RING  RUN',///)                                 AMB0218
  300 CONTINUE                                                           AMB0219
      STOP                                                               AMB0220
      END                                                                AMB0221
      SUBROUTINE INIDAT                                                  AMB0222
      IMPLICIT REAL*8(A-H,O-Z)                                           AMB0223
      REAL*8 LAMDA0,LAMDA1                                               AMB0224
      CHARACTER*8 XNAME,XNAMED                                           AMB0225
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB0226
     1            G16,G17,G18,G19,G20                                    AMB0227
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,      AMB0228
     1           DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                       AMB0229
      COMMON /NPAR/NPHI,IPAR,NP,NR,NX,NXS,NS,NSPEC,NS1,NS2,NTAU0,NETA0,  AMB0230
     1            NAMB,NCASE,ICASE,IFAN                                 AMB0231
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,     AMB0232
     1            CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB0233
     2            PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0,  AMB0234
     3            DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)        AMB0235
      COMMON /EPSIL/EPSETA,EPST,EPSR                                     AMB0236
      COMMON /EXTREM/TEXT(5),ETAEXT(5),ETAKXT(5),PHIEXT(5),              AMB0237
     1               PSIEXT(5),EMEXT(5),FEXT(5),WEXT(5),                 AMB0238
     2               TMAX(5),ETAKMX(5),ETAMAX(5),PSIMAX(5),              AMB0239
     3               EMMAX(5),FMAX(5),                                   AMB0240
     4               RFMAX(5),PHIFMX(5),PHIMAX(5),WMAX(5)                AMB0241
      COMMON /COUNTS/ICONTC,ICONTT,ICNTOT,ICNTMX,IQTOT(5),ISHAD(5)       AMB0242
      COMMON /SPEC/WAV,XC(5),WC(5),WRC(5),XNAME(5),QFC(5),QDC(5),        AMB0243
     1            QU2C(5),FLUXC(5),OMEGA(5),FLXU2C(5),URMSC(5)          AMB0244
      COMMON /AMBIEN/ENA,UA,PSIA,PHIA,HA(3),WA(3),                       AMB0245
     1               UAX,UAY,UAZ,AA,BA,CA,RA,XA,YA,ZA,SHADOW             AMB0246
      COMMON /POINT/XP,YP,XCOR,YCOR                                      AMB0247
      LOGICAL SHADOW                                                     AMB0248
      DIMENSION XCD(5),WCD(5),XNAMED(5)                                  AMB0249
      DATA XCD/.091D0,.091D0,.104D0,.135D0,.579D0/                       AMB0250
      DATA WCD/1.00D0,20.0D0,2.00D0,21.0D0,4.00D0/                       AMB0251
      DATA XNAMED/'   H  ','  HF  ','  H2  ','  DF  ','  HE  '/         AMB0252
      DATA IFIRST/0/                                                     AMB0253
      IFAN=2                                                             AMB0254
      PAI=4.D0*DATAN(1.D0)                                               AMB0255
      PAI2=PAI/2.D0                                                      AMB0256
      DEG=180.D0/PAI                                                     AMB0257
      AR=8.3143D3                                                        AMB0258
      AV=6.022D 26                                                       AMB0259
C  OMEGAC=0.5 IS FOR HARD SPHERE COLLISIONS,                             AMB0260
C  AN AVERAGE RECOMMENDED VALUE IS ABOUT OMEGAC=0.75                     AMB0261
      OMEGAC=0.5D0                                                       AMB0262
      NSPEC=5                                                            AMB0263
      NS1=2                                                              AMB0264
      NS2=2                                                              AMB0265
      DO 51 NS=1,NSPEC                                                   AMB0266
      XC(NS)=XCD(NS)                                                     AMB0267
      WC(NS)=WCD(NS)                                                     AMB0268
      XNAME(NS)=XNAMED(NS)                                               AMB0269
   51 CONTINUE                                                           AMB0270
C  COMBINE HF AND DF MOLE FRACTIONS INTO HF FRACTION                     AMB0271
      XC(2)=XC(2)+XC(4)                                                  AMB0272
      XC(4)=0.                                                           AMB0273
C                                                                        AMB0274
      A0=2.5D0                                                           AMB0275
      EM1=4.D0                                                           AMB0276
      RH00=0.0075D0                                                      AMB0277
      T0=1400.D0                                                         AMB0278
      G=1.54D0                                                           AMB0279
      D=2.5D-10                                                          AMB0280
      NXS=1                                                              AMB0281
      XSI=1.0D0                                                          AMB0282
      XSF=10.D0                                                          AMB0283
C  AMBIENT AIR                                                           AMB0284
      ENA=1.00D 16                                                       AMB0285
      UA=8.D 3                                                           AMB0286
      NAMB=3                                                             AMB0287
      WA(1)=28.D0                                                        AMB0288
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      WA(2)=32.D0                                                        AMB0289
      WA(3)=16.D0                                                        AMB0290
      HA(1)=1.D0                                                         AMB0291
      HA(2)=0.                                                           AMB0292
      HA(3)=0.                                                           AMB0293
      PSIA=20.D0/DEG                                                     AMB0294
      PHIA=0.00D0/DEG                                                    AMB0295
C  INTEGRATION PARAMETERS                                                AMB0296
      NPHI=6                                                             AMB0297
      NTAU0=4                                                            AMB0298
      NETA0=4                                                            AMB0299
      ICNTMX=100                                                         AMB0300
      RMIN=0.                                                            AMB0301
      DR0=0.10D0                                                         AMB0302
      DPSI0=0.20D0                                                       AMB0303
      DTMAX=1.0D0                                                        AMB0304
      DETA0=0.50D0                                                       AMB0305
      ETALIM=10.D0                                                       AMB0306
      EPST=0.5D0                                                         AMB0307
      EPSR=0.3D0                                                         AMB0308
      FACT=1.D 20                                                        AMB0309
      RETURN                                                             AMB0310
C*********************************************************************  AMB0311
C  COMPUTATION OF DATA-DEPENDENT PARAMETERS                              AMB0312
C*********************************************************************  AMB0313
      ENTRY INDAT1                                                       AMB0314
C*********************************************************************  AMB0315
      ALOGF=DLOG(FACT)                                                   AMB0316
      WAV=0.                                                             AMB0317
      DO 52 NS=1,NSPEC                                                   AMB0318
      WAV=WAV+XC(NS)*WC(NS)                                              AMB0319
 52   CONTINUE                                                           AMB0320
      DO 53 NS=1,NSPEC                                                   AMB0321
      WRC(NS)=WC(NS)/WAV                                                 AMB0322
 53   CONTINUE                                                           AMB0323
      SIGMA=PAI*D**2                                                     AMB0324
      EN0=RHO0*AV/WAV                                                    AMB0325
      C0=DSQRT(G*AR*T0/WAV)                                              AMB0326
      XSV(1)=XSI                                                         AMB0327
      IF(NXS.EQ.1) GO TO 12                                              AMB0328
      DXL=(DLOG(XSF)-DLOG(XSI))/(DBLE(NXS)-1.D0)                         AMB0329
      XLI=DLOG(XSI)                                                      AMB0330
      DO 11 NX=2,NXS                                                     AMB0331
      XSV(NX)=DEXP(XLI+(DBLE(NX)-1.D0)*DXL)                              AMB0332
 11   CONTINUE                                                           AMB0333
 12   CONTINUE                                                           AMB0334
      G1=(G-1.D0)/2.D0                                                   AMB0335
      G2=(G+1.D0)/(2.D0*(G-1.D0))                                        AMB0336
      G3=G/2.D0                                                          AMB0337
      G4=(G+1.D0)/(G-1.D0)                                               AMB0338
      G5=DSQRT((G+1.D0)/(G-1.D0))                                        AMB0339
      G6=1.D0/(G-1.D0)                                                   AMB0340
      G7=2.D0/(G+1.D0)                                                   AMB0341
      G8=(0.5D0*(G+1.D0)**2/(G-1.D0))**(1.D0/(G+1.D0))*                  AMB0342
     1   ((G+1.D0)/(G-1.D0))**((G-1.D0)/(G+1.D0))                        AMB0343
      G9=(G+3.D0)/(2.D0*(G-1.D0))                                        AMB0344
      G10=(7.D0-3.D0*G)/(2.D0*(G-1.D0))                                  AMB0345
      G11=(5.D0-3.D0*G)/(2.D0*(G-1.D0))                                  AMB0346
      G13=(2.D0-G)/(2.D0*(G-1.D0))                                       AMB0347
      G14=G/(2.D0*(G-1.D0))                                              AMB0348
      G15=(G+1.D0)/(3.D0-G)                                              AMB0349
      ZETA1=G5*DATAN(DSQRT(EM1**2-1.D0)/G5)                              AMB0350
      AMU1=DASIN(1.D0/EM1)                                               AMB0351
      PSI1=PAI2+AMU1                                                     AMB0352
      SPSI1=DSIN(PSI1)                                                   AMB0353
      CPSI1=DCOS(PSI1)                                                   AMB0354
      PSIF=PAI2+AMU1+ZETA1-G5*PAI2                                       AMB0355
      SPSIF=DSIN(PSIF)                                                   AMB0356
      CPSIF=DCOS(PSIF)                                                   AMB0357
      TPSIF=DTAN(PSIF)                                                   AMB0358
      TETA1=PSI1-AMU1                                                    AMB0359
      ST1=DSIN(TETA1)                                                    AMB0360
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      CT1=DCOS(TETA1)                                                    AMB0361
      Q0=ENA*UA                                                          AMB0362
      LAMDA0=1.D0/(DSQRT(2.D0)*SIGMA*EN0)                                AMB0363
      LAMDA1=LAMDA0*(1.D0+G1*EM1**2)**(G6-OMEGAC+0.5D0)                  AMB0364
      AA=DCOS(PSIA)                                                      AMB0365
      BA=DSIN(PSIA)*DCOS(PHIA)                                           AMB0366
      CA=DSIN(PSIA)*DSIN(PHIA)                                           AMB0367
      UAX=-UA*AA                                                         AMB0368
      UAY=-UA*BA                                                         AMB0369
      UAZ=-UA*CA                                                         AMB0370
      XCOR=0.                                                            AMB0371
      YCOR=A0                                                            AMB0372
C                                                                        AMB0373
      PRINT 201,NSPEC,XNAME                                              AMB0374
 201  FORMAT(/1X,'SPECIES DATA   NSPEC=',I3/                             AMB0375
     1       1X,'SPECIES NAMES    ',11(2X,A6,2X))                       AMB0376
      PRINT 202,XC                                                       AMB0377
 202  FORMAT( 1X,'MOLE FRACTION XC=',11(F8.4,2X))                        AMB0378
      PRINT 203,WC                                                       AMB0379
 203  FORMAT( 1X,'MOL. WEIGHT   WC=',11(F8.4,2X))                        AMB0380
      PRINT 21,AR,AV,WAV,G,RHO0,T0,EN0,C0,D                              AMB0381
 21   FORMAT(/1X,'THERMODYNAMIC DATA'/                                   AMB0382
     1       1X,'AR,AV,WAV,GAMMA=',2X,2D14.5,2F9.3/                      AMB0383
     2       1X,'RHO0,T0,EN0,C0,D=',D12.4,F8.0,D13.5,2D12.4)             AMB0384
      PRINT 22,EM1,PSI1*DEG,PSIF*DEG,                                    AMB0385
     1         A0,LAMDA0,LAMDA1                                          AMB0386
 22   FORMAT(/1X,'FLOW AND GEOMETRY DATA'/                               AMB0387
     1       1X,'EM1,PSI1,PSIF=',3F9.3/                                  AMB0388
     2       1X,'A0,LAMDA0,LAMDA1=',F9.3,2D13.4)                         AMB0389
      PRINT 23,DPSI0,DTMAX,DETA0,ETALIM,DR0,RMIN,                        AMB0390
     1         EPST,EPSR,                                                AMB0391
     2         NPHI,NTAU0,NETA0                                          AMB0392
 23   FORMAT(/1X,'INTEGRATION DATA'/                                     AMB0393
     1       1X,'DPSI0,DTMAX,DETA0,ETALIM=',4F9.4/                       AMB0394
     2       1X,'DR0,RMIN,=',2D13.4/                                     AMB0395
     3       1X,'EPST,EPSR=',2D12.3/                                     AMB0396
     4       1X,'NPHI,NTAU0,NETA0=',3I6)                                 AMB0397
      PRINT 24,ENA,UA,PSIA*DEG,PHIA*DEG                                  AMB0398
 24   FORMAT(/1X,'ABBREVIATED AIR DATA'/                                 AMB0399
     1       1X,'ENA,UA=',2D13.4/                                        AMB0400
     2       1X,'PSIA,PHIA=',2F9.1)                                      AMB0401
      GO TO (251,252), IFAN                                              AMB0402
                                                                         AMB0403
 251  CONTINUE                                                           AMB0404
      PRINT 2510, IFAN                                                   AMB0405
 2510 FORMAT(/1X,'RING-FAN APPROXIMATED AS PLANAR.   IFAN=',I4)          AMB0406
      GO TO 250                                                          AMB0407
 252  CONTINUE                                                           AMB0408
      PRINT 2520, IFAN                                                   AMB0409
 2520 FORMAT(/1X,'RING-FAN APPROXIMATED BY MATCHED APPROXIMATION.',      AMB0410
     1       4X,'IFAN=',I4)                                              AMB0411
 250  CONTINUE                                                           AMB0412
      PRINT 29                                                           AMB0413
 29   FORMAT(///1X,'END  DATA'///)                                       AMB0414
      IF(IFIRST.EQ.0.AND.IFAN.EQ.2)                                      AMB0415
     1CALL HMSET                                                         AMB0416
      IF(IFAN.EQ.2) IFIRST=IFIRST+1                                      AMB0417
      RETURN                                                             AMB0418
      END                                                                AMB0419
C$OPTIONS LIST                                                           AMB0420
      SUBROUTINE SOF(ISTOP)                                              AMB0421
      IMPLICIT REAL*8(A-H,O-Z)                                           AMB0422
      CHARACTER*4 ISTOP(1)                                               AMB0423
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB0424
     1            G16,G17,G18,G19,G20                                    AMB0425
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,      AMB0426
     1           DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                        AMB0427
      COMMON /NPAR/NPHI,IPAR,NP,NR,NX,NXS,NS,NSPEC,NS1,NS2,NTAU0,NETA0,  AMB0428
     1            NAMB,NCASE,ICASE,IFAN                                  AMB0429
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,     AMB0430
     1            CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB0431
     2            PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0,  AMB0432
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     3              DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)          AMB0433
      COMMON /EPSIL/EPSETA,EPST,EPSR                                        AMB0434
      COMMON /EXTREM/TEXT(5),ETAEXT(5),ETAKXT(5),PHIEXT(5),                 AMB0435
     1               PSIEXT(5),EMEXT(5),FEXT(5),WEXT(5),                   AMB0436
     2               TMAX(5),ETAKMX(5),ETAMAX(5),PSIMAX(5),                AMB0437
     3               EMMAX(5),FMAX(5),                                     AMB0438
     4               RFMAX(5),PHIFMX(5),PHIMAX(5),WMAX(5)                  AMB0439
      COMMON /SOFPR/C,DSUMF,DSUMD,T,ETA,DETA,SUM,DSUM,SUMU,DSUMU            AMB0440
      COMMON /SUMS/SUMF(5),SUMD(5),SUMU2(5)                                 AMB0441
      COMMON /COUNTS/ICONTC,ICONTT,ICNTOT,ICNTMX,IQTOT(5),ISHAD(5)          AMB0442
      COMMON /SPEC/WAV,XC(5),WC(5),WRC(5),XNAME(5),QFC(5),QDC(5),           AMB0443
     1               QU2C(5),FLUXC(5),OMEGA(5),FLXU2C(5),URMSC(5)          AMB0444
      PRINT 1,ISTOP                                                         AMB0445
    1 FORMAT(///1X,2H**,2X,30A4,2X,2H**,///)                                AMB0446
      PRINT 71,NS,NP,NR,NX,ICONTC,ICONTT                                    AMB0447
   71 FORMAT(1X,'NS,NP,NR,NX,ICONTC,ICONTT=',6I6/)                          AMB0448
      IF(NS.GT.NSPEC) NS=1                                                  AMB0449
      PRINT 72,RF,PHIF*DEG,PHISOF*DEG,W*DEG,BETA*DEG                        AMB0450
   72 FORMAT(/1X,'RF,PHIF,PHISOF,W,BETA=',D14.5,4F10.3/)                    AMB0451
      PRINT 73,C,T,TLIM,ETA                                                 AMB0452
   73 FORMAT(/1X,'C,T,TLIM,ETA=',4D14.5/)                                   AMB0453
      PRINT 74,DSUM,SUM,DSUMF,SUMF(NS),SUMD(NS),QDC(NS),QFC(NS),            AMB0454
     1         FLUXC(NS),OMEGA(NS)                                          AMB0455
   74 FORMAT(1X,'DSUM,SUM,DSUMF,SUMF(NS),SUMD(NS)=',5D14.5/                 AMB0456
     1       1X,'QDC(NS),QFC(NS),FLUXC(NS),OMEGA(NS)=',4D14.5/)             AMB0457
      XX=-1.D0                                                              AMB0458
      YY=DSQRT(XX)+1.D0                                                     AMB0459
      STOP                                                                  AMB0460
      END                                                                   AMB0461
      SUBROUTINE FLUX                                                       AMB0462
      IMPLICIT REAL*8(A-H,O-Z)                                              AMB0463
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB0464
     1             G16,G17,G18,G19,G20                                      AMB0465
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,         AMB0466
     1            DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                        AMB0467
      COMMON /NPAR/NPHI,IPAR,NP,NR,NX,NXS,NS,NSPEC,NS1,NS2,NTAU0,NETA0,     AMB0468
     1             NAMB,NCASE,ICASE,IFAN                                    AMB0469
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,        AMB0470
     1             CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB0471
     2             PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0,   AMB0472
     3             DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)         AMB0473
      COMMON /EPSIL/EPSETA,EPST,EPSR                                        AMB0474
      COMMON /EXTREM/TEXT(5),ETAEXT(5),ETAKXT(5),PHIEXT(5),                 AMB0475
     1               PSIEXT(5),EMEXT(5),FEXT(5),WEXT(5),                   AMB0476
     2               TMAX(5),ETAKMX(5),ETAMAX(5),PSIMAX(5),                AMB0477
     3               EMMAX(5),FMAX(5),                                     AMB0478
     4               RFMAX(5),PHIFMX(5),PHIMAX(5),WMAX(5)                  AMB0479
      COMMON /SOFPR/C,DSUMF,DSUMD,T,ETA,DETA,SUM,DSUM,SUMU,DSUMU            AMB0480
      COMMON /COUNTS/ICONTC,ICONTT,ICNTOT,ICNTMX,IQTOT(5),ISHAD(5)          AMB0481
      COMMON /SPEC/WAV,XC(5),WC(5),WRC(5),XNAME(5),QFC(5),QDC(5),           AMB0482
     1             QU2C(5),FLUXC(5),OMEGA(5),FLXU2C(5),URMSC(5)           AMB0483
      COMMON /SUMS/SUMF(5),SUMD(5),SUMU2(5)                                 AMB0484
      EL0=SIGMA*RF*EN0                                                      AMB0485
      IF(Z0.NE.0.)                                                          AMB0486
     1CALL SOF('THE SCHEME HERE IS NOT WRITTEN FOR Z0.NE.0.')               AMB0487
      YY0=(Y0-A0)/X0                                                        AMB0488
      PCHECK=DATAN(YY0)                                                     AMB0489
      IF(PCHECK.GT.PSIF-1.D-4.OR.PCHECK.LT.-1.D-4)                          AMB0490
     1CALL SOF('FLUX RECEIVING POINT WITHIN FAN OR WITHIN SPACECRAFT')      AMB0491
      SPHIF=DSIN(PHIF)                                                      AMB0492
      CPHIF=DCOS(PHIF)                                                      AMB0493
      XF=RF*CPSIF                                                           AMB0494
      YF=APF*CPHIF                                                          AMB0495
      ZF=APF*SPHIF                                                          AMB0496
      TBETA=ZF/(YF-Y0)                                                      AMB0497
      BETA=DATAN(TBETA)                                                     AMB0498
      IF(DABS(BETA).GT.PAI2) CALL SOF('BETA.GT.PAI/2')                      AMB0499
      SBETA=DSIN(BETA)                                                      AMB0500
      CBETA=DCOS(BETA)                                                      AMB0501
      DIST=DSQRT((XF-X0)**2+(YF-Y0)**2+(ZF-Z0)**2)                          AMB0502
      CW=(XF-X0)/DIST                                                       AMB0503
      SW=DSQRT(1.D0-CW**2)                                                  AMB0504
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      W=PAI2-DATAN(CW/SW)                                                AMB0505
      OMEGX=CW                                                           AMB0506
      OMEGY=SW*CBETA                                                     AMB0507
      OMEGZ=SW*SBETA                                                     AMB0508
      CALL LIMIT                                                         AMB0509
C                                                                        AMB0510
      DO 20 NS=NS1,NS2                                                   AMB0511
      SUMF(NS)=0.                                                        AMB0512
      SUMU2(NS)=0.                                                       AMB0513
      SUMD(NS)=0.                                                        AMB0514
      FEXT(NS)=0.                                                        AMB0515
      CALL SUMT                                                          AMB0516
      SUMF(NS)=SUM                                                       AMB0517
      SUMU2(NS)=SUMU                                                     AMB0518
      QFC(NS)=SUMF(NS)/FACT                                              AMB0519
      QU2C(NS)=SUMU2(NS)/FACT                                            AMB0520
      FEXT(NS)=FEXT(NS)/FACT                                             AMB0521
      CALL FAN(TEXT(NS),PSIEXT(NS),PHIEXT(NS))                          AMB0522
      IF(PSIEXT(NS).LT.PSIF-1.D-10) CALL SOF('PSIEXT(NS).LT.PSIF')       AMB0523
      IF(PSIEXT(NS).GT.PSI1) PSIEXT(NS)=PSI1                             AMB0524
      PSI0=PSIEXT(NS)                                                    AMB0525
      T=TEXT(NS)                                                         AMB0526
      CALL MATCH(T,PSI0,EM,TETA)                                         AMB0527
      EMEXT(NS)=EM                                                       AMB0528
      WEXT(NS)=W                                                         AMB0529
      IQTOT(NS)=IQTOT(NS)+ICONTT                                         AMB0530
 20   CONTINUE                                                           AMB0531
      RETURN                                                             AMB0532
      END                                                                AMB0533
      SUBROUTINE LIMIT                                                   AMB0534
      IMPLICIT REAL*8(A-H,O-Z)                                           AMB0535
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB0536
     1            G16,G17,G18,G19,G20                                    AMB0537
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,      AMB0538
     1           DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                       AMB0539
      COMMON /NPAR/NPHI,IPAR,NP,NR,NX,NXS,NS,NSPEC,NS1,NS2,NTAU0,NETA0,  AMB0540
     1            NAMB,NCASE,ICASE,IFAN                                  AMB0541
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,     AMB0542
     1            CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB0543
     2            PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0,  AMB0544
     3            DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)         AMB0545
      COMMON /EPSIL/EPSETA,EPST,EPSR                                     AMB0546
      COMMON /EXTREM/TEXT(5),ETAEXT(5),ETAKXT(5),PHIEXT(5),              AMB0547
     1               PSIEXT(5),EMEXT(5),FEXT(5),WEXT(5),                 AMB0548
     2               TMAX(5),ETAKMX(5),ETAMAX(5),PSIMAX(5),              AMB0549
     3               EMMAX(5),FMAX(5),                                   AMB0550
     4               RFMAX(5),PHIFMX(5),PHIMAX(5),WMAX(5)                AMB0551
      COMMON /SPEC/WAV,XC(5),WC(5),WRC(5),XNAME(5),QFC(5),QDC(5),        AMB0552
     1            QU2C(5),FLUXC(5),OMEGA(5),FLXU2C(5),URMSC(5)           AMB0553
      AAA=(CW/CPSI1)**2-1.D0                                             AMB0554
      IF(AAA.LT.1.D-10) GO TO 1                                          AMB0555
      TPSI1=SPSI1/CPSI1                                                  AMB0556
      AP1=A0+XF*TPSI1                                                    AMB0557
      BBB=2.D0*(AP1*CW*TPSI1-SW*APF*(CBETA*CPHIF+SBETA*SPHIF))           AMB0558
      CCC=AP1**2-APF**2                                                  AMB0559
      DDD=BBB**2-4.D0*AAA*CCC                                            AMB0560
      TLIM=(-BBB+DSQRT(DDD))/(2.D0*AAA)                                  AMB0561
      TLIM=TLIM/RF                                                       AMB0562
      RETURN                                                             AMB0563
 1    CONTINUE                                                           AMB0564
      TLIM=1.D 55                                                        AMB0565
      RETURN                                                             AMB0566
      END                                                                AMB0567
      SUBROUTINE SUMT                                                    AMB0568
      IMPLICIT REAL*8(A-H,O-Z)                                           AMB0569
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB0570
     1            G16,G17,G18,G19,G20                                    AMB0571
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,      AMB0572
     1           DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                       AMB0573
      COMMON /NPAR/NPHI,IPAR,NP,NR,NX,NXS,NS,NSPEC,NS1,NS2,NTAU0,NETA0,  AMB0574
     1            NAMB,NCASE,ICASE,IFAN                                  AMB0575
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,     AMB0576
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     1             CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB0577
     2             PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0,  AMB0578
     3             DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)        AMB0579
      COMMON /EPSIL/EPSETA,EPST,EPSR                                     AMB0580
      COMMON /EXTREM/TEXT(5),ETAEXT(5),ETAKXT(5),PHIEXT(5),              AMB0581
     1               PSIEXT(5),EMEXT(5),FEXT(5),WEXT(5),                 AMB0582
     2               TMAX(5),ETAKMX(5),ETAMAX(5),PSIMAX(5),              AMB0583
     3               EMMAX(5),FMAX(5),                                   AMB0584
     4               RFMAX(5),PHIFMX(5),PHIMAX(5),WMAX(5)                AMB0585
      COMMON /SOFPR/CC,DSUMF,DSUMD,T,ETA,DETA,SUM,DSUM,SUMU,DSUMU        AMB0586
      COMMON /COUNTS/ICONTC,ICONTT,ICNTOT,ICNTMX,IQTOT(5),ISHAD(5)       AMB0587
      COMMON /SPEC/WAV,XC(5),WC(5),WRC(5),XNAME(5),QFC(5),QDC(5),        AMB0588
     1            QU2C(5),FLUXC(5),OMEGA(5),FLXU2C(5),URMSC(5)          AMB0589
      COMMON /SUMS/SUMF(5),SUMD(5),SUMU2(5)                              AMB0590
C  INTEGRATION OF FLUX ARRIVING ALONG A SINGLE RAY                       AMB0591
      DT=DPSI0                                                           AMB0592
      PSIN=PSIF                                                          AMB0593
      ETA1=0.                                                            AMB0594
      ETA3=0.                                                            AMB0595
      FETA4=0.                                                           AMB0596
      FETAU4=0.                                                          AMB0597
      T=0.                                                               AMB0598
      SUM=0.                                                             AMB0599
      SUMU=0.                                                            AMB0600
      ICONTT=0                                                           AMB0601
    1 ICONTT=ICONTT+1                                                    AMB0602
      PSIL=PSIN                                                          AMB0603
      DT2=DT/2.D0                                                        AMB0604
      DT6=DT/6.D0                                                        AMB0605
      T1=T+DT2                                                           AMB0606
      T2=T+DT                                                            AMB0607
      FETA1=FETA4                                                        AMB0608
      FETAU1=FETAU4                                                      AMB0609
      CALL PATHK(T1,ETAK1)                                               AMB0610
      CALL FETA(T1,ETA1,ETAK1,GT2,FETA2,FETAU2)                          AMB0611
      FETA3=FETA2                                                        AMB0612
      FETAU3=FETAU2                                                      AMB0613
      CALL PATHK(T2,ETAK3)                                               AMB0614
      CALL FETA(T2,ETA3,ETAK3,GT4,FETA4,FETAU4)                          AMB0615
      DETA=DT*GT4                                                        AMB0616
      DSUM=DT6*(FETA1+2.D0*(FETA2+FETA3)+FETA4)                          AMB0617
      DSUMU=DT6*(FETAU1+2.D0*(FETAU2+FETAU3)+FETAU4)                     AMB0618
      T=T+DT                                                             AMB0619
      ETA=ETA3                                                           AMB0620
      ETAK=ETAK3                                                         AMB0621
      SUM=SUM+DSUM                                                       AMB0622
      SUMU=SUMU+DSUMU                                                    AMB0623
      IF(FEXT(NS).GT.FETA4) GO TO 10                                     AMB0624
      FEXT(NS)=FETA4                                                     AMB0625
      TEXT(NS)=T                                                         AMB0626
      ETAEXT(NS)=ETA                                                     AMB0627
      ETAKXT(NS)=ETAK                                                    AMB0628
   10 CONTINUE                                                           AMB0629
C  STEP CONTROL (DT)                                                     AMB0630
      CALL FAN(T,PSI,PHI)                                                AMB0631
      IF(PSI.LT.PSIF-1.D-10) CALL SOF('PSI.LT.PSIF')                     AMB0632
      IF(PSI.GT.PSI1) PSI=PSI1                                           AMB0633
      PSIN=PSI                                                           AMB0634
      DPSI=PSIN-PSIL                                                     AMB0635
      DTP=DT*(DPSI0/(DPSI+1.D-10))                                       AMB0636
      DTE=DT*(DETA0/(DETA+1.D-10))                                       AMB0637
      DT1=1.2D0*DT                                                       AMB0638
      DT=DMIN1(DTP,DTE,DT1,DTMAX)                                        AMB0639
      IF(DT.LE.0.) CALL SOF('COMPUTED DT NEGATIVE')                      AMB0640
   15 CONTINUE                                                           AMB0641
      IF(IPAR.LT.1)                                                      AMB0642
     1PRINT 111,NR,NP,T,PSI*DEG,PHI*DEG,ETA,ETAK,SUM,DSUM/(SUM+1.D-20)  AMB0643
  111  FORMAT(1X,'NR,NP,T,PSI,PHI=',2I3,3D12.3/                          AMB0644
     1       1X,'ETA,ETAK,SUM,ERRR=',4D12.3)                             AMB0645
      IF(ICONTT.GT.ICNTMX)                                               AMB0646
     1CALL SOF('ICONTT TOO LARGE')                                       AMB0647
      IF(ICONTT.LE.2) GO TO 1                                            AMB0648
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      IF(ETA+ETAK.GT.ETALIM) GO TO 100                                  AMB0649
      IF(T.GT.50.D0 .OR. T*RF.GT.A0) GO TO 100                          AMB0650
      IF(SUM.EQ.0.) GO TO 1                                             AMB0651
      ERR=(DSUM/SUM)/DT                                                 AMB0652
      IF(ERR.GT.EPST) GO TO 1                                           AMB0653
  100 CONTINUE                                                          AMB0654
      SUM=SUM*EL0                                                       AMB0655
      SUMU=SUMU*EL0                                                     AMB0656
      RETURN                                                            AMB0657
      END                                                               AMB0658
      SUBROUTINE FETA(T,ETAIK,ETAK,GT,FET,FETU2)                        AMB0659
      IMPLICIT REAL*8(A-H,O-Z)                                          AMB0660
      REAL*8 MU1,MU2                                                    AMB0661
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB0662
     1            G16,G17,G18,G19,G20                                   AMB0663
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,     AMB0664
     1           DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                      AMB0665
      COMMON /NPAR/NPHI,IPAR,NP,NR,NX,NXS,NS,NSPEC,NS1,NS2,NTAU0,NETA0, AMB0666
     1            NAMB,NCASE,ICASE,IFAN                                 AMB0667
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,    AMB0668
     1            CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB0669
     2            PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0, AMB0670
     3            DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)        AMB0671
      COMMON /EPSIL/EPSETA,EPST,EPSR                                    AMB0672
      COMMON /EXTREM/TEXT(5),ETAEXT(5),ETAKXT(5),PHIEXT(5),             AMB0673
     1               PSIEXT(5),EMEXT(5),FEXT(5),WEXT(5),                AMB0674
     2               TMAX(5),ETAKMX(5),ETAMAX(5),PSIMAX(5),             AMB0675
     3               EMMAX(5),FMAX(5),                                  AMB0676
     4               RFMAX(5),PHIFMX(5),PHIMAX(5),WMAX(5)               AMB0677
      COMMON /SPEC/WAV,XC(5),WC(5),WRC(5),XNAME(5),QFC(5),QDC(5),       AMB0678
     1            QU2C(5),FLUXC(5),OMEGA(5),FLXU2C(5),URMSC(5)          AMB0679
      COMMON /AMBIEN/ENA,UA,PSIA,PHIA,HA(3),WA(3),                      AMB0680
     1               UAX,UAY,UAZ,AA,BA,CA,RA,XA,YA,ZA,SHADOW            AMB0681
      LOGICAL SHADOW                                                    AMB0682
      COMMON /NAGESH/PIK,UIK,UIKX,UIKY,UIKZ                             AMB0683
      ETAIK=0.                                                          AMB0684
      IF(SHADOW) GO TO 1                                                AMB0685
      K=1                                                               AMB0686
      I=NS                                                              AMB0687
      CALL FAN(T,PSI,PHI)                                               AMB0688
      IF(PSI.LT.PSIF-1.D-10) CALL SOF('PSI.LT.PSIF')                    AMB0689
      IF(PSI.GT.PSI1) PSI=PSI1                                          AMB0690
      PSI0=PSI                                                          AMB0691
      CALL MATCH(T,PSI0,EM,TETA)                                        AMB0692
      SPSI=DSIN(PSI)                                                    AMB0693
      CPSI=DCOS(PSI)                                                    AMB0694
      SPHI=DSIN(PHI)                                                    AMB0695
      CPHI=DCOS(PHI)                                                    AMB0696
      ST=DSIN(TETA)                                                     AMB0697
      CT=DCOS(TETA)                                                     AMB0698
      GOREM=1.D0+G1*EM**2                                               AMB0699
      TERMN=GOREM**G6                                                   AMB0700
      U=EM*C0/DSQRT(GOREM)                                              AMB0701
      UX=U*CT                                                           AMB0702
      UY=U*ST*CPHI                                                      AMB0703
      UZ=U*ST*SPHI                                                      AMB0704
C     COLLISION                                                         AMB0705
      MU1=WC(I)/(WC(I)+WA(K))                                           AMB0706
      MU2=1.D0-MU1                                                      AMB0707
      UMX=MU1*UX+MU2*UAX                                                AMB0708
      UMY=MU1*UY+MU2*UAY                                                AMB0709
      UMZ=MU1*UZ+MU2*UAZ                                                AMB0710
      DOTUM=OMEGX*UMX+OMEGY*UMY+OMEGZ*UMZ                               AMB0711
      URX=UX-UAX                                                        AMB0712
      URY=UY-UAY                                                        AMB0713
      URZ=UZ-UAZ                                                        AMB0714
      UR=DSQRT(URX**2+URY**2+URZ**2)                                    AMB0715
      DET=DOTUM**2+(MU2*UR)**2-(UMX**2+UMY**2+UMZ**2)                   AMB0716
      IF(DET.LT.0.) GO TO 1                                             AMB0717
      DET1=DSQRT(DET)                                                   AMB0718
      UIK1=-DOTUM+DET1                                                  AMB0719
      UIK2=-DOTUM-DET1                                                  AMB0720
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      IF(UIK2.GT.0.) CALL SOF('DOUBLE COLLISION OPTION NOT PROGRAMMED   AMB0721
     1 YET')                                                            AMB0722
      UIK=UIK1                                                          AMB0723
      IF(UIK.LE.0.) GO TO 1                                             AMB0724
      UIKX=-OMEGX*UIK                                                   AMB0725
      UIKY=-OMEGY*UIK                                                   AMB0726
      UIKZ=-OMEGZ*UIK                                                   AMB0727
      CDEL=(DOTUM+UIK)/(MU2*UR)                                         AMB0728
      IF(CDEL.LE.0.) CALL SOF('CDEL NEGATIVE NOT PROGRAMMED YET')       AMB0729
      IF(CDEL-1.D-10.GT.1.D0)                                           AMB0730
     1CALL SOF('CDEL (COS(DELTA)) CANNOT BE GT.1.')                     AMB0731
      PIK=(UIK/(MU2*UR))**2/(4.D0*PAI*CDEL)                             AMB0732
      IF (PIK.LT.0.) CALL SOF('PIK.LT.0')                               AMB0733
      FET=(UR/UA)*PIK/TERMN                                             AMB0734
      UREL=DSQRT((UX-UIKX)**2+(UY-UIKY)**2+(UZ-UIKZ)**2)                AMB0735
      GT=EL0*(UREL/UIK)/TERMN                                           AMB0736
      CALL PATHIK(T,ETAIK)                                              AMB0737
      POWER=ETAIK+ETAK-ALOGF                                            AMB0738
      EFACT=0.                                                          AMB0739
      IF(POWER.LT.60.D0)EFACT=DEXP(-POWER)                              AMB0740
      FET=FET*EFACT                                                     AMB0741
      FETU2=FET*UIK**2                                                  AMB0742
      IF(EM.LT.0.) CALL SOF('EM.LT.0')                                  AMB0743
      FETU2=FET*EM                                                      AMB0744
      RETURN                                                            AMB0745
    1 CONTINUE                                                          AMB0746
      FET=0.                                                            AMB0747
      FETU2=0.                                                          AMB0748
      GT=0.                                                             AMB0749
      RETURN                                                            AMB0750
      END                                                               AMB0751
      SUBROUTINE PATHIK(TC,ETAIK)                                       AMB0752
      IMPLICIT REAL*8(A-H,O-Z)                                          AMB0753
      REAL*8 MU1,MU2                                                    AMB0754
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB0755
     1            G16,G17,G18,G19,G20                                   AMB0756
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,     AMB0757
     1           DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                       AMB0758
      COMMON /NPAR/NPHI,IPAR,NP,NR,NX,NXS,NS,NSPEC,NS1,NS2,NTAU0,NETA0, AMB0759
     1            NAMB,NCASE,ICASE,IFAN                                 AMB0760
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,    AMB0761
     1            CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB0762
     2            PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0, AMB0763
     3            DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)        AMB0764
      COMMON /EPSIL/EPSETA,EPST,EPSR                                    AMB0765
      COMMON /EXTREM/TEXT(5),ETAEXT(5),ETAKXT(5),PHIEXT(5),             AMB0766
     1               PSIEXT(5),EMEXT(5),FEXT(5),WEXT(5),                AMB0767
     2               TMAX(5),ETAKMX(5),ETAMAX(5),PSIMAX(5),             AMB0768
     3               EMMAX(5),FMAX(5),                                  AMB0769
     4               RFMAX(5),PHIFMX(5),PHIMAX(5),WMAX(5)               AMB0770
      COMMON /SPEC/WAV,XC(5),WC(5),WRC(5),XNAME(5),QFC(5),QDC(5),       AMB0771
     1            QU2C(5),FLUXC(5),OMEGA(5),FLXU2C(5),URMSC(5)          AMB0772
      COMMON /AMBIEN/ENA,UA,PSIA,PHIA,HA(3),WA(3),                      AMB0773
     1               UAX,UAY,UAZ,AA,BA,CA,RA,XA,YA,ZA,SHADOW            AMB0774
      LOGICAL SHADOW                                                    AMB0775
      NETA=NETA0                                                        AMB0776
      DT=TC/DBLE(NETA)                                                  AMB0777
      DT2=DT/2.D0                                                       AMB0778
      DT6=DT/6.D0                                                       AMB0779
      GT4=0.                                                            AMB0780
      T=0.                                                              AMB0781
      ETA=0.                                                            AMB0782
      IT=0                                                              AMB0783
    1 IT=IT+1                                                           AMB0784
      T1=T+DT2                                                          AMB0785
      T2=T+DT                                                           AMB0786
      GT1=GT4                                                           AMB0787
      CALL FT(T1,GT2)                                                   AMB0788
      GT3=GT2                                                           AMB0789
      CALL FT(T2,GT4)                                                   AMB0790
      DETA=DT6*(GT1+2.D0*(GT2+GT3)+GT4)                                 AMB0791
      T=T+DT                                                            AMB0792
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      ETA=ETA+DETA                                                      AMB0793
      IF(IT.LT.NETA) GO TO 1                                            AMB0794
      ETAIK=ETA                                                         AMB0795
      RETURN                                                            AMB0796
      END                                                               AMB0797
      SUBROUTINE FT(T,GT)                                               AMB0798
      IMPLICIT REAL*8(A-H,O-Z)                                          AMB0799
      REAL*8 MU1,MU2                                                    AMB0800
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB0801
     1            G16,G17,G18,G19,G20                                   AMB0802
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,     AMB0803
     1           DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                       AMB0804
      COMMON /NPAR/NPHI,IPAR,NP,NR,NX,NXS,NS,NSPEC,NS1,NS2,NTAU0,NETA0, AMB0805
     1            NAMB,NCASE,ICASE,IFAN                                 AMB0806
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,    AMB0807
     1            CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB0808
     2            PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0, AMB0809
     3            DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)        AMB0810
      COMMON /EPSIL/EPSETA,EPST,EPSR                                    AMB0811
      COMMON /EXTREM/TEXT(5),ETAEXT(5),ETAKXT(5),PHIEXT(5),             AMB0812
     1               PSIEXT(5),EMEXT(5),FEXT(5),WEXT(5),                AMB0813
     2               TMAX(5),ETAKMX(5),ETAMAX(5),PSIMAX(5),             AMB0814
     3               EMMAX(5),FMAX(5),                                  AMB0815
     4               RFMAX(5),PHIFMX(5),PHIMAX(5),WMAX(5)               AMB0816
      COMMON /SPEC/WAV,XC(5),WC(5),WRC(5),XNAME(5),QFC(5),QDC(5),       AMB0817
     1            QU2C(5),FLUXC(5),OMEGA(5),FLXU2C(5),URMSC(5)          AMB0818
      COMMON /AMBIEN/ENA,UA,PSIA,PHIA,HA(3),WA(3),                      AMB0819
     1               UAX,UAY,UAZ,AA,BA,CA,RA,XA,YA,ZA,SHADOW            AMB0820
      LOGICAL SHADOW                                                    AMB0821
      COMMON /NAGESH/PIK,UIK,UIKX,UIKY,UIKZ                             AMB0822
      K=1                                                               AMB0823
      I=NS                                                              AMB0824
      CALL FAN(T,PSI,PHI)                                               AMB0825
      IF(PSI.LT.PSIF-1.D-10) CALL SOF('PSI.LT.PSIF')                    AMB0826
      IF(PSI.GT.PSI1) PSI=PSI1                                          AMB0827
      PSI0=PSI                                                          AMB0828
      CALL MATCH(T,PSI0,EM,TETA)                                        AMB0829
      SPSI=DSIN(PSI)                                                    AMB0830
      CPSI=DCOS(PSI)                                                    AMB0831
      SPHI=DSIN(PHI)                                                    AMB0832
      CPHI=DCOS(PHI)                                                    AMB0833
      ST=DSIN(TETA)                                                     AMB0834
      CT=DCOS(TETA)                                                     AMB0835
      GOREM=1.D0+G1*EM**2                                               AMB0836
      TERMN=GOREM**G6                                                   AMB0837
      U=EM*C0/DSQRT(GOREM)                                              AMB0838
      UX=U*CT                                                           AMB0839
      UY=U*ST*CPHI                                                      AMB0840
      UZ=U*ST*SPHI                                                      AMB0841
      UREL=DSQRT((UX-UIKX)**2+(UY-UIKY)**2+(UZ-UIKZ)**2)                AMB0842
      GT=EL0*(UREL/UIK)/TERMN                                           AMB0843
      RETURN                                                            AMB0844
      END                                                               AMB0845
      SUBROUTINE PATHK(T,ETAK)                                          AMB0846
      IMPLICIT REAL*8(A-H,O-Z)                                          AMB0847
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB0848
     1            G16,G17,G18,G19,G20                                   AMB0849
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,     AMB0850
     1           DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                       AMB0851
      COMMON /NPAR/NPHI,IPAR,NP,NR,NX,NXS,NS,NSPEC,NS1,NS2,NTAU0,NETA0, AMB0852
     1            NAMB,NCASE,ICASE,IFAN                                 AMB0853
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,    AMB0854
     1            CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB0855
     2            PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0, AMB0856
     3            DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)        AMB0857
      COMMON /EPSIL/EPSETA,EPST,EPSR                                    AMB0858
      COMMON /EXTREM/TEXT(5),ETAEXT(5),ETAKXT(5),PHIEXT(5),             AMB0859
     1               PSIEXT(5),EMEXT(5),FEXT(5),WEXT(5),                AMB0860
     2               TMAX(5),ETAKMX(5),ETAMAX(5),PSIMAX(5),             AMB0861
     3               EMMAX(5),FMAX(5),                                  AMB0862
     4               RFMAX(5),PHIFMX(5),PHIMAX(5),WMAX(5)               AMB0863
      COMMON /COUNTS/ICONTC,ICONTT,ICNTOT,ICNTMX,IQTOT(5),ISHAD(5)      AMB0864
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      COMMON /AMBIEN/ENA,UA,PSIA,PHIA,HA(3),WA(3),                      AMB0865
     1             UAX,UAY,UAZ,AA,BA,CA,RA,XA,YA,ZA,SHADOW              AMB0866
      LOGICAL SHADOW                                                    AMB0867
      ETAK=0.                                                           AMB0868
C  DETERMINE POINT OF ENTRY OF AMBIENT TRAJECTORY TO FAN                AMB0869
      TRF=T*RF                                                          AMB0870
      XC=XF+TRF*OMEGX                                                   AMB0871
      YC=YF+TRF*OMEGY                                                   AMB0872
      ZC=ZF+TRF*OMEGZ                                                   AMB0873
C  CHECK SHADOW                                                         AMB0874
      SHADOW=.FALSE.                                                    AMB0875
      EVER=BA**2+CA**2                                                  AMB0876
      DETS=EVER*A0**2-(BA*ZC-CA*YC)**2                                  AMB0877
      IF(DETS.LE.0.) GO TO 2                                            AMB0878
      DETS1=DSQRT(DETS)                                                 AMB0879
      TAU1=(-(BA*YC+CA*ZC)+DETS1)/EVER                                  AMB0880
      IF(TAU1.GT.0.) SHADOW=.TRUE.                                      AMB0881
 2    CONTINUE                                                          AMB0882
      IF(SHADOW) GO TO 10                                               AMB0883
      EVER1=A0+XC*TPSIF                                                 AMB0884
      EVER2=BA**2+CA**2-(AA*TPSIF)**2                                   AMB0885
      EVER3=BA*YC+CA*ZC-AA*EVER1*TPSIF                                  AMB0886
      DET=EVER3**2-EVER2*(YC**2+ZC**2-EVER1**2)                         AMB0887
      IF(DET.LE.0.)                                                     AMB0888
     1CALL SOF('NO INTERSECTION OF AMB. TRAJ. WITH LIMITING CONE')      AMB0889
      DET1=DSQRT(DET)                                                   AMB0890
      TAUP=(-EVER3+DET1)/EVER2                                          AMB0891
      TAUM=(-EVER3-DET1)/EVER2                                          AMB0892
      IF(TAUP.GT.0. .AND. TAUM.GT.0.)                                   AMB0893
     1CALL SOF('TWO POSITIVE INTERSECTIONS WITH LIMITING CONE.NOT PERMITAMB0894
     1 IN THIS VERSION')                                                AMB0895
      TAUF=DMAX1(TAUP,TAUM)                                             AMB0896
      IF(TAUF.LE.0.)                                                    AMB0897
     1CALL SOF('NO POSITIVE INTERSECTION WITH LIMITING CONE')           AMB0898
      XA=XC+TAUF*AA                                                     AMB0899
      YA=YC+TAUF*BA                                                     AMB0900
      ZA=ZC+TAUF*CA                                                     AMB0901
      RA=DSQRT(XA**2+(DSQRT(YA**2+ZA**2)-A0)**2)                        AMB0902
      TAUF=TAUF/RA                                                      AMB0903
      NTAU=NTAU0                                                        AMB0904
      DTAU=TAUF/DBLE(NTAU)                                              AMB0905
      ETAK=0.                                                           AM30906
      TAU=0.                                                            AMB0907
      DTAU2=DTAU/2.D0                                                   AMB0908
      DTAU6=DTAU/6.D0                                                   AMB0909
      GTAU4=0.                                                          AMB0910
      ITAU=0                                                            AMB0911
 1    ITAU=ITAU+1                                                       AMB0912
      TAU1=TAU+DTAU2                                                    AMB0913
      TAU2=TAU+DTAU                                                     AMB0914
      GTAU1=GTAU4                                                       AMB0915
      CALL FTAU(TAU1,GTAU2)                                             AMB0916
      GTAU3=GTAU2                                                       AMB0917
      CALL FTAU(TAU2,GTAU4)                                             AMB0918
      DETAK=DTAU6*(GTAU1+2.D0*(GTAU2+GTAU3)+GTAU4)                      AMB0919
      TAU=TAU+DTAU                                                      AMB0920
      ETAK=ETAK+DETAK                                                   AMB0921
      IF(ITAU.LT.NTAU) GO TO 1                                          AMB0922
      ETAK=ETAK*(SIGMA*EN0*RA)                                          AMB0923
      RETURN                                                            AMB0924
 10   CONTINUE                                                          AMB0925
      ISHAD(NS)=ISHAD(NS)+1                                             AMD0926
      RETURN                                                            AMB0927
      END                                                               AMB0928
      SUBROUTINE FTAU(TAU,GTAU)                                         AMB0929
      IMPLICIT REAL*8(A-H,O-Z)                                          AMB0930
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB0931
     1            G16,G17,G18,G19,G20                                   AMB0932
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,     AMB0933
     1          DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                      AMB0934
      COMMON /NPAR/NPHI,IPAR,NP,NR,NX,NXS,NS,NSPEC,NS1,NS2,NTAU0,NETA0, AMB0935
     1            NAMB,NCASE,ICASE,IFAN                                 AMB0936
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FILE: AMB SCRIPT A1

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      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,     AMB0937
     1             CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB0938
     2             PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0,  AMB0939
     3             DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)        AMB0940
      COMMON /EPSIL/EPSETA,EPST,EPSR                                     AMB0941
      COMMON /EXTREM/TEXT(5),ETAEXT(5),ETAKXT(5),PHIEXT(5),              AMB0942
     1               PSIEXT(5),EMEXT(5),FEXT(5),WEXT(5),                 AMB0943
     2               TMAX(5),ETAKMX(5),ETAMAX(5),PSIMAX(5),              AMB0944
     3               EMMAX(5),FMAX(5),                                   AMB0945
     4               RFMAX(5),PHIFMX(5),PHIMAX(5),WMAX(5)                AMB0946
      COMMON /SPEC/WAV,XC(5),WC(5),WRC(5),XNAME(5),QFC(5),QDC(5),        AMB0947
     1             QU2C(5),FLUXC(5),OMEGA(5),FLXU2C(5),URMSC(5)          AMB0948
      COMMON /AMBIEN/ENA,UA,PSIA,PHIA,HA(3),WA(3),                       AMB0949
     1               UAX,UAY,UAZ,AA,BA,CA,RA,XA,YA,ZA,SHADOW             AMB0950
      LOGICAL SHADOW                                                     AMB0951
      CALL FANT(TAU,PSI,PHI)                                             AMB0952
      IF(PSI.LT.PSIF-1.D-10) CALL SOF('PSI.LT.PSIF')                     AMB0953
      IF(PSI.GT.PSI1) PSI=PSI1                                           AMB0954
      PSI0=PSI                                                           AMB0955
      CALL MATCH(T,PSI0,EM,TETA)                                         AMB0956
      SPSI=DSIN(PSI)                                                     AMB0957
      CPSI=DCOS(PSI)                                                     AMB0958
      SPHI=DSIN(PHI)                                                     AMB0959
      CPHI=DCOS(PHI)                                                     AMB0960
      ST=DSIN(TETA)                                                      AMB0961
      CT=DCOS(TETA)                                                      AMB0962
      GOREM=1.D0+G1*EM**2                                                AMB0963
      TERMN=GOREM**G6                                                    AMB0964
      U=EM*C0/DSQRT(GOREM)                                               AMB0965
      UREL=DSQRT((CT*U-UAX)**2+(ST*CPHI*U-UAY)**2+(ST*SPHI*U-UAZ)**2)    AMB0966
      GTAU=UREL/(UA*TERMN)                                               AMB0967
      RETURN                                                             AMB0968
      END                                                                AMB0969
      SUBROUTINE FAN(T,PSI,PHI)                                          AMB0970
      IMPLICIT REAL*8(A-H,O-Z)                                           AMB0971
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,      AMB0972
     1            DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                       AMB0973
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,     AMB0974
     1             CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB0975
     2             PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0,  AMB0976
     3             DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)        AMB0977
      COMMON /POINT/XP,YP,XCOR,YCOR                                      AMB0978
C  RING FAN GEOMETRY.   FAN CORNER IS AT (0,A0*COS(PHI),A0*SIN(PHI)).    AMB0979
C  RF  --  RADIAL DISTANCE ON LIMITING CHARACTERISTIC OF POINT OF        AMB0980
C          ENTRANCE OF RAY.                                             AMB0981
C  DIRECTION COSINES OF RAY: OMEGX,OMEGY,OMEGZ                          AMB0982
      TRF=T*RF                                                           AMB0983
      X=XF+TRF*OMEGX                                                     AMB0984
      Y=YF+TRF*OMEGY                                                     AMB0985
      Z=ZF+TRF*OMEGZ                                                     AMB0986
      DY=DSQRT(Y*Y+Z*Z)-A0                                               AMB0987
      IF(DABS(DY).LE.1.D-10*A0) DY=1.D-10*A0                             AMB0988
      IF(DY.LT.0.)                                                       AMB0989
     1CALL SOF('POINT X,Y,X CANNOT BE CLOSER TO X-AXIS THAN RADIUS A0')  AMB0990
      YY=X/DY                                                            AMB0991
      PSI=PAI2-DATAN(YY)                                                 AMB0992
      PHI=DATAN(Z/Y)                                                     AMB0993
      XP=XCOR+X                                                          AMB0994
      YP=A0+DY                                                           AMB0995
      RETURN                                                             AMB0996
      END                                                                AMB0997
      SUBROUTINE FANT(TAU,PSI,PHI)                                       AMB0998
      IMPLICIT REAL*8(A-H,O-Z)                                           AMB0999
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,      AMB1000
     1            DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                       AMB1001
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,     AMB1002
     1             CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB1003
     2             PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0,  AMB1004
     3             DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)        AMB1005
      COMMON /AMBIEN/ENA,UA,PSIA,PHIA,HA(3),WA(3),                       AMB1006
     1               UAX,UAY,UAZ,AA,BA,CA,RA,XA,YA,ZA,SHADOW             AMB1007
      COMMON /POINT/XP,YP,XCOR,YCOR                                      AMB1008
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      LOGICAL SHADOW                                                    AMB1009
C  RING FAN GEOMETRY.   FAN CORNER IS AT (0,A0*COS(PHI),A0*SIN(PHI)).   AMB1010
C  RA  --  RADIAL DISTANCE ON LIMITING CHARACTERISTIC OF POINT OF       AMB1011
C          ENTRANCE OF RAY.                                             AMB1012
C  DIRECTION COSINES OF RAY: -AA,-BA,-CA                                AMB1013
      TRA=TAU*RA                                                        AMB1014
      X=XA-TRA*AA                                                       AMB1015
      Y=YA-TRA*BA                                                       AMB1016
      Z=ZA-TRA*CA                                                       AMB1017
      DY=DSQRT(Y*Y+Z*Z)-A0                                              AMB1018
      IF(DABS(DY).LE.1.D-10*A0) DY=1.D-10*A0                            AMB1019
      IF(DY.LT.0.)                                                      AMB1020
     1CALL SOF('POINT X,Y,X CANNOT BE CLOSER TO X-AXIS THAN RADIUS A0') AMB1021
      YY=X/DY                                                           AMB1022
      PSI=PAI2-DATAN(YY)                                                AMB1023
      PHI=DATAN(Z/Y)                                                    AMB1024
      XP=XCOR+X                                                         AMB1025
      YP=A0+DY                                                          AMB1026
      RETURN                                                            AMB1027
      END                                                               AMB1028
      SUBROUTINE HMSET                                                  AMB1029
C  SUBROUTINE NUMBER  20                                                AMB1030
      IMPLICIT REAL*8(A-H,O-Z,$)                                        AMB1031
      REAL*8 KAPA0B,MHINV,MINV0,M,MF,M1,M2,M3,NORM,MEXIT,LAMD0B         AMB1032
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB1033
     1            G16,G17,G18,G19,G20                                   AMB1034
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,     AMB1035
     1           DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                       AMB1036
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,    AMB1037
     1            CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB1038
     2            PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0,  AMB1039
     3            DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)        AMB1040
      COMMON /GRP/DMINV,MHINV(101),HMV(101)                             AMB1041
      COMMON /IGRP/KHM                                                  AMB1042
C  A ROUTINE FOR THE C+ DERIVATIVE DUE TO RING SYMMETRY (GRP).          AMB1043
      MEXIT=EM1                                                         AMB1044
      KHM=51                                                            AMB1045
      IF(KHM.GT.101) CALL SOF('2001')                                   AMB1046
      MINV0=1.D0/MEXIT                                                  AMB1047
      DMINV=MINV0/DBLE(KHM-1)                                           AMB1048
      M=MEXIT                                                           AMB1049
      SUM=0.                                                            AMB1050
      KHM1=KHM-1                                                        AMB1051
      DO 1 I=1,KHM1                                                     AMB1052
      MF=M                                                              AMB1053
      MHINV(I)=MINV0-DBLE(I-1)*DMINV                                    AMB1054
      M=1.D0/MHINV(I)                                                   AMB1055
      DM=M-MF                                                           AMB1056
      M1=M-DM                                                           AMB1057
      M2=M-DM/2.D0                                                      AMB1058
      M3=M                                                              AMB1059
      CALL MFUNC(M1,F1,ETALF1,TETA1)                                    AMB1060
      CALL MFUNC(M2,F2,ETALF2,TETA2)                                    AMB1061
      CALL MFUNC(M3,F3,ETALF3,TETA3)                                    AMB1062
      SUM=SUM+DM*(F1+4.D0*F2+F3)/6.D0                                   AMB1063
      ETALF=ETALF3                                                      AMB1064
      TETA=TETA3                                                        AMB1065
      PSI=TETA+DASIN(1.D0/M)                                            AMB1066
      NORM=((3.D0-G)/4.D0)*(M**2-1.D0)**0.75D0/                         AMB1067
     1     (DSIN(PSI)*(1.D0+G1*M**2)**G14)                              AMB1068
      HM=SUM*NORM                                                       AMB1069
      HMV(I)=HM                                                         AMB1070
      GOREM=1.D0+G1*M**2                                                AMB1071
      GOR=M**2-1.D0                                                     AMB1072
      DELT0B=0.5D0*DSQRT(GOR)*(1.D0/(MEXIT*ETALF)                       AMB1073
     1       +DSIN(TETA)/M)/DSIN(PSI)+G15*HM/2.D0                       AMB1074
      EPSI0B=DELT0B/DSQRT(GOR)-DSIN(TETA)/(M*DSIN(PSI))                 AMB1075
      KAPA0B=1.D0                                                       AMB1076
      IF(DABS(PAI2-TETA).GT.1.D-6)                                      AMB1077
     1KAPA0B=DTAN(TETA)*EPSI0B                                          AMB1078
      LAMD0B=EPSI0B-DELT0B*GOREM/(GOR*DSQRT(GOR))                       AMB1079
      PRINT 11,I,M,HM,TETA*DEG,PSI*DEG                                  AMB1080
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 11   FORMAT(/1X,'            I,M,HM,TETA,PSI=',I5,5D12.4)                    AMB1081
      PRINT 12,DELTOB,EPSIOB*DEG,KAPAOB*DEG,LAMDOB*DEG                        AMB1082
 12   FORMAT( 1X,'DELTOB,EPSIOB,KAPAOB,LAMDOB=',5X,5D12.4)                   AMB1083
 1    CONTINUE                                                                AMB1084
      MHINV(KHM)=0.                                                           AMB1085
      HMV(KHM)=1.D0                                                           AMB1086
      RETURN                                                                  AMB1087
      END                                                                     AMB1088
      SUBROUTINE MFUNC(M,F,ETALF,TETA)                                        AMB1089
C  SUBROUTINE NUMBER  21                                                      AMB1090
      IMPLICIT REAL*8(A-H,O-Z,$)                                              AMB1091
      REAL*8 NU,NUFUNC,M,MEXIT,MD,MDD                                         AMB1092
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB1093
     1           G16,G17,G18,G19,G20                                          AMB1094
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,       AMB1095
     1          DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                            AMB1096
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,      AMB1097
     1           CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB1098
     2           PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0, AMB1099
     3           DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)          AMB1100
C                                                                             AMB1101
      QF(MDD)=1.D0/DSQRT(MDD**2-1.D0)                                         AMB1102
      NUFUNC(MD)=-G5*DATAN(G5*QF(MD))+DATAN(QF(MD))                           AMB1103
C                                                                             AMB1104
      MEXIT=EM1                                                               AMB1105
      NU=NUFUNC(M)                                                            AMB1106
      TETA=NUFUNC(MEXIT)+PAI2-NU                                              AMB1107
      GOREM=1.D0+G1*M**2                                                      AMB1108
      GOR=M**2-1.D0                                                           AMB1109
      F=(M**2)*(GOREM**G13)*DSIN(TETA)/GOR**1.25D0                            AMB1110
      GOREM1=1.D0+G1*MEXIT**2                                                 AMB1111
      GOR1=MEXIT**2-1.D0                                                      AMB1112
      ETALF=((GOREM/GOREM1)**G14)*((GOR1/GOR)**0.25D0)                        AMB1113
      RETURN                                                                  AMB1114
      END                                                                     AMB1115
      SUBROUTINE HINTER(M,H)                                                  AMB1116
C  SUBROUTINE NUMBER  22                                                      AMB1117
      IMPLICIT REAL*8(A-H,O-Z,$)                                              AMB1118
      REAL*8 MINV,M,MEXIT,MHINV                                               AMB1119
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB1120
     1           G16,G17,G18,G19,G20                                          AMB1121
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,       AMB1122
     1          DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                            AMB1123
      COMMON /GRP/DMINV,MHINV(101),HMV(101)                                   AMB1124
      COMMON /IGRP/KHM                                                        AMB1125
C COMPUTE H(M) BY INTERPOLATION                                               AMB1126
      MEXIT=EM1                                                               AMB1127
      MINV=1.D0/M                                                             AMB1128
      I=KHM-IDINT(MINV/DMINV-1.D-9)-1                                         AMB1129
      IF(I.GE.1.AND.I.LT.KHM) GO TO 1                                         AMB1130
      PRINT 11,I,KHM,M,MEXIT                                                  AMB1131
 11   FORMAT(/1X,'I,KHM,M,MEXIT=',2I5,2D14.6/)                               AMB1132
      CALL SOF('2201')                                                        AMB1133
 1    CONTINUE                                                                AMB1134
      F1=(MINV-MHINV(I+1))/DMINV                                              AMB1135
      F2=1.D0-F1                                                              AMB1136
      IF(F1.LT.-1.D-9) CALL SOF('2210')                                       AMB1137
      IF(F2.LT.-1.D-9) CALL SOF('2211')                                       AMB1138
      H=F1*HMV(I)+F2*HMV(I+1)                                                 AMB1139
      RETURN                                                                  AMB1140
      END                                                                     AMB1141
      SUBROUTINE MATCH(T,PSI0,MAB,TETAAB)                                     AMB1142
C  SUBROUTINE NUMBER  23                                                      AMB1143
      IMPLICIT REAL*8(A-H,O-Z,$)                                              AMB1144
      REAL*8 M,M0B,MEXIT,MAB,LAMDOB,KAPAOB                                    AMB1145
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB1146
     1           G16,G17,G18,G19,G20                                          AMB1147
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,       AMB1148
     1          DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                            AMB1149
      COMMON /NPAR/NPHI,IPAR,NP,NR,NX,NXS,NS,NSPEC,NS1,NS2,NTAU0,NETA0,   AMB1150
     1           NAMB,NCASE,ICASE,IFAN                                       AMB1151
      COMMON /GEOM/APF,PAI,PAI2,W,SW,CW,BETA,SBETA,CBETA,PSI1,SPSI1,      AMB1152
```

```
     1             CPSI1,PSIF,SPSIF,CPSIF,TPSIF,AK,SK,CK,A0,RF,XF,YF,ZF,AMB1153
     2             PHISOF,PHIF,SPHIF,CPHIF,DYMIN,RMIN,XS,DIST,X0,Y0,Z0, AMB1154
     3             DY0,DEG,PSIN,ST1,CT1,OMEGX,OMEGY,OMEGZ,XSV(21)       AMB1155
      COMMON /POINT/XP,YP,XCOR,YCOR                                     AMB1156
      COMMON /GRP/DMINV,MHINV(101),HMV(101)                             AMB1157
      COMMON /IGRP/KHM                                                  AMB1158
      MEXIT=EM1                                                         AMB1159
      GO TO (101,102),IFAN                                              AMB1160
 101  CONTINUE                                                          AMB1161
C  FAN APPROXIMATED AS PLANAR                                          AMB1162
      MAB=DSQRT(1.D0+G4/DTAN((PSI(  PSIF)/G5)**2)                       AMB1163
      TETAAB=PSI0-DASIN(1.D0/MAB)                                       AMB1164
      GO TO 100                                                         AMB1165
 102  CONTINUE                                                          AMB1166
C  COMPUTE MAB FROM THE INVERSE PROBLEM SOLUTION                       AMB1167
      COTAV=1.D0/DTAN(PSI0)                                             AMB1168
      EVY=YP*DLOG(YP/YCOR)/(YP-YCOR)-1.D0                               AMB1169
      PSIN=PSI0                                                         AMB1170
      DO 1 ITER=1,10                                                    AMB1171
      PSI=PSIN                                                          AMB1172
      M=DSQRT(1.D0+G4/DTAN((PSI-PSIF)/G5)**2)                           AMB1173
      M=DMAX1(M,MEXIT)                                                  AMB1174
      CALL HINTER(M,HM)                                                 AMB1175
      CALL MFUNC(M,F,ETALF,TETA)                                        AMB1176
      GOREM=1.D0+G1*M**2                                                AMB1177
      GOR=M**2-1.D0                                                     AMB1178
      DELTOB=0.5D0*DSQRT(GOR)*(1.D0/(MEXIT*ETALF)                       AMB1179
     1      +DSIN(TETA)/M)/DSIN(PSI)+G15*HM/2.D0                        AMB1180
      EPSIOB=DELTOB/DSQRT(GOR)-DSIN(TETA)/(M*DSIN(PSI))                 AMB1181
      LAMDOB=EPSIOB-DELTOB*GOREM/(GOR*DSQRT(GOR))                       AMB1182
      COTN=COTAV+LAMDOB*EVY/DSIN(PSI)**2                                AMB1183
      PSIN=PAI2-DATAN(COTN)                                             AMB1184
      DPSI=PSIN-PSI                                                     AMB1185
      IF(DABS(DPSI).LT.1.D-6) GO TO 11                                  AMB1186
   1  CONTINUE                                                          AMB1187
      PRINT 12,I,ITER,PSI,PSIN,DPSI,M,XP,YP,T                           AMB1188
  12  FORMAT(/1X,'I,ITER,PSI,PSIN,DPSI,M,XP,YP,T='//                   AMB1189
     1       1X,2I4,7D11.3/)                                           AMB1190
      CALL SOF('2301')                                                  AMB1191
  11  CONTINUE                                                          AMB1192
C  USING M0B=M AS COMPUTED FROM THE INVERSE PROBLEM, FIND MAB.         AMB1193
      M0B=M                                                             AMB1194
      CALL MFUNC(M,F,ETALF,TETA)                                        AMB1195
      PSI=TETA+DASIN(1.D0/M)                                            AMB1196
      CALL HINTER(M,HM)                                                 AMB1197
      GOREM=1.D0+G1*M**2                                                AMB1198
      GOR=M**2-1.D0                                                     AMB1199
      DELTOB=0.5D0*DSQRT(GOR)*(1.D0/(MEXIT*ETALF)                       AMB1200
     1      +DSIN(TETA)/M)/DSIN(PSI)+G15*HM/2.D0                        AMB1201
      F0B=(G7*GOREM)**G2/M                                              AMB1202
      FAB=F0B*(YP/YCOR)**DELTOB                                         AMB1203
      CALL AREAF(FAB,MAB)                                               AMB1204
      EPSIOB=DELTOB/DSQRT(GOR)-DSIN(TETA)/(M*DSIN(PSI))                 AMB1205
      KAPA0B=1.D0                                                       AMB1206
      IF(DABS(PAI2-TETA).GT.1.D-8)                                      AMB1207
     1KAPA0B=DTAN(TETA)*EPSIOB                                          AMB1208
      COSTAB=DCOS(TETA)*(YP/YCOR)**(-KAPA0B)                            AMB1209
      TETAAB=DACOS(COSTAB)                                              AMB1210
 100  CONTINUE                                                          AMB1211
      RETURN                                                            AMB1212
      END                                                               AMB1213
      SUBROUTINE AREAF(F,M)                                             AMB1214
C  SUBROUTINE NUMBER  24                                               AMB1215
      IMPLICIT REAL*8(A-H,O-Z,$)                                        AMB1216
      REAL*8 MEXIT,MIN,M,MHINV                                          AMB1217
      COMMON /GAMA/G,G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12,G13,G14,G15,AMB1218
     1             G16,G17,G18,G19,G20                                  AMB1219
      COMMON /PAR/C0,EN0,EM1,D,SIGMA,TLIM,DR0,EL0,Q0,T0,FACT,ALOGF,     AMB1220
     1            DPSI0,DTMAX,DETA0,ETALIM,XSI,XSF                      AMB1221
      COMMON /GRP/DMINV,MHINV(101),HMV(101)                             AMB1222
      COMMON /IGRP/KHM                                                  AMB1223
C  COMPUTE MACH NUMBER M FROM AREA RATIO FUNCTION F                     AMB1224
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C  F=((2/(G+1))*(1+(G-1)*M**2))**((G+1)/(2*(G-1)))/M                    AMB1225
C  INITIAL GUESS IS MIN                                                 AMB1226
      MEXIT=EM1                                                        AMB1227
      E1=(F*MEXIT)**(1.D0/G2)/G7                                       AMB1228
      E2=(E1-1.D0)/G1                                                  AMB1229
      E3=DMAX1(E2,MEXIT**2)                                            AMB1230
      MIN=DSQRT(E3)                                                    AMB1231
      EMN=MIN                                                          AMB1232
      DO 1 I=1,100                                                     AMB1233
      EMO=EMN                                                          AMB1234
      GOREM=1.D0+G1*EMO**2                                             AMB1235
      GOR=EMO**2-1.D0                                                  AMB1236
      FO=(C7*GOREM)**G2/EMO                                            AMB1237
      DF=FO-F                                                          AMB1238
C     PRINT 123,I,EMO,EMN,FO,F,DF,GOR,GOREM                            AMB1239
C123  FORMAT(1X,'I,EMO,EMN,FO,F,DF,GOR,GOREM=',I5,7D12.4)              AMB1240
      DFDM=FO*GOR/(EMO*GOREM)                                          AMB1241
      DMN=DF/DFDM                                                      AMB1242
      EMN=EMO-DMN                                                      AMB1243
      EPSEM=DABS(DMN/EMN)                                              AMB1244
      IF(EPSEM.LT.1.D-10) GO TO 11                                     AMB1245
 1    CONTINUE                                                         AMB1246
      CALL SOF('2401')                                                 AMB1247
 11   CONTINUE                                                         AMB1248
      M=EMN                                                            AMB1249
      RETURN                                                           AMB1250
      END                                                              AMB1251
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